

## Stability Theory Methods in Mechanics Problems

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The research is concerned with various aspects of qualitative analysis in Complex systems dynamics, including fundamental problems of modelling in Mechanics. Generalized approach, based on Lyapunov's stability theory and Chetayev's idea, is developed. Besides the investigated objects are treated for unified view point on formed basic postulates (stability and singularity) as singularly perturbed ones.

It offers ample scope to obtain the reduction principle analogue, valid for general qualitative analysis and synthesis problems; to establish the effective technique for problems of decomposition in Mechanics. The concepts and methods of classical stability theory are developed for problems of singularly perturbed systems; the statements of N.G.Chetayev and P.A.Kuzmin are extended for stability problems with parametric perturbations in non-regular case. It enables to investigate the critical and transcendental cases, inherent to mechanics systems.

The non-trivial cases are considered here: the unperturbed system is on the boundary of stability domain; the generating system is singular one (non-traditional approximate  $s$ -system, that is non-generated, non-limit one); the original system is decomposed on a few subsystems, that are describing the behaviour of different-frequency variables, in accordance with multiple time scales.

This approach is combining the manners of stability theory and perturbations theory, with extending statements of I.M.Gradstein, A.N.Tikhonov. The correctness problems of constructed shortened models are solved as singularly perturbed problems (via understanding the problems of modelling as stability problems under singular parametric perturbations). In this way:

- the regular manners for constructing optimal shortened mechanic-mathematical models ( $s$ -models as comparison systems) are worked out;
- the qualitative equivalence conditions are obtained, with corresponding estimations, valid on infinite time interval, both in regular-nominal cases and in particular-singular cases;
- the effective scheme is developed for decomposition procedure both for original model and for original dynamic properties, including stability property.

Besides the hierarchy of state variables is established by natural way automatically; the sequences of nonlinear comparison systems are built in accordance with hierarchic structure of variables; the correspondence between original model and shortened one is revealed.

The received results are generalizing and supplementing ones, known in theory of perturbations; these results are developing interesting applications in engineering. With reference to Mechanics the rigorous theoretic justification is obtained for considered approximate models and theories, both traditional

(K.Magnus, A.Andronov, D.Merkin,...) and new ones (in particular, inertialess model, precessional model, Aristotle's model of point mass dynamics,...).

The work is developing approximate methods for nonlinear analysis and synthesis in large-scale systems dynamics. A.M.Lyapunov methodology, N.G.Chetayev stability postulate, K.P.Persidskiy quasi-stability postulate combined with asymptotic approach allow to establish the effective method as additional activity tool for OR in problems of modelling of complex systems, qualitative analysis, control, synthesis. Constructed approach, founded on stability/singularity postulates, is creating optimal systemic method for fundamental problems in dynamic systems, with subsystems of different nature (natural- scientific, social-political,..), with strong substantiation of approximate theories. It is important for our Knowledge in whole .It is corresponding to -

**Antonio Gaudi** points ,brilliant nonlinear analyst –  
“ **Science is Analysis and Synthesis**”;  
“ **Nature is my Teacher always...**”

and to –

**V.I.Vernadskiy** points (**V.I.Vernadskiy**, RAS Academician ):

“...*We specialize not on Sciences, but on Problems. The Problems these do not pack in frames single, determined, established Science...*”;  
“... *It is **empirical generalizations**, which are the acknowledging fact, not having for it the explanation...*”.

In any research it is necessary understand : **mathematics gives for us power tool**; but -

“Mathematics – it is **the mill** which will **grind** the all that is put on its millstones ....”

And from these view points the solving dynamic problems of complex multi-scale systems in Mechanics is requiring the higher professionalism in whole; which is provided by A.M.Lyapunov theory methodology.

We show it:

for any real dynamic system -

**MODELLING is ART ....**

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