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Smoothing with Measurement Information Retrodictions in Cluttered Environments

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Abstract—**This paper presents a smoothing data association algorithm for a single target tracking in clutter. The proposed algorithm fuses the forward estimates and all the available measurement information retrodictions (but not backward track estimates) within the smoothing window to obtain the smoothed estimates. The measurement information retrodictions are obtained using the one-step-backward information filter propagation for fast calculation. The simulation studies show that the proposed algorithm improves the false track discrimination performance with similar root mean squared errors as the existing smoothing integrated probabilistic data association algorithm.**

Keywords—**Smoothing data association, measurement information retrodictions, false track discrimination.**

I. Introduction

Smoothing [1]–[2] is used to obtain the deferred state estimates at scan *k* with a batch of measurements including the future measurements. This can result in a better trajectory estimation, as well as false track discrimination (FTD) performance because more measurements are available at the cost of deferred estimation. These improvements are highly effective in applications such as "threat assessment," and "situation awareness"[3].

We consider the problem of smoothing data association for target tracking in clutter. This class of problem has received considerable attention in the literature. Rauch-Tung-Striebel (RTS) [2] smoothing is used in probabilistic multi-hypothesis tracking (PMHT) [4]. multi-scan multi-hypothesis tracking smoothing [5] applies RTS trajectory smoothing, ignoring the FTD functionality. Based on integrated probabilistic data association (IPDA) [6] Augmented-state IPDA (ASIPDA) [7] calculates the probability of target existence as a track quality measure. Smoothing IPDA (sIPDA) [8] uses the RTS smoothing formulae to calculate smoothing predictions and smoothing innovations. sIPDA delivers tangible improvements in the FTD capabilities over both IPDA and ASIPDA. But the weakness of the algorithm is that the track must be retained until the end of the smoothing window to obtain the smoothed estimates because all the updated states and predicted states within the smoothing window are required for the RTS formulas.

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In this paper, we present a smoothing data association algorithm using the measurement information retrodictions to overcome the weakness of sIPDA. We name the proposed algorithm "smoothing integrated probabilistic data association with information retrodictions (sIPDA-IR)." sIPDA-IR fuses the forward updated estimates from the forward IPDA [6] and the measurement information retrodictions within the smoothing window, iteratively to obtain the smoothed track state and target existence probability (PTE). As the smoothing data association is performed from *k*+1 to *N(>k)* iteratively, the estimation error of the smoothed estimates is reduced. The measurement information retrodiction from scan *N* to scan *k* can be obtained using the one-step-backward information filter propagation function. Due to the singularity of the inverse covariance, matrix diagonalization in [9] is applied to calculate the likelihood functions of the measurement information retrodictions for data association. A comparative assessment is carried out to verify the FTD benefit of the proposed algorithm.

II. Models

The usual fundamental assumptions apply:

1) *Infinite sensor resolution*. Each measurement has one source, either the target or clutter.

2) *A point target*. Each target creates zero or one detection per measurement time (scan).

A. Target Model

The target trajectory state $\mathbf{x}_{k-1}$ is an $n\times 1$ vector at scan *k*-1 and propagates as

$$\mathbf{x}_k = \mathbf{F}_{\Delta T}\mathbf{x}_{k-1} + \mathbf{v}_{\Delta T} \tag{1}$$

where $\mathbf{F}_{\Delta T}$ denotes the state propagation matrix with constant time interval ΔT, and the zero mean and white Gaussian plant noise sequence $\mathbf{v}_{\Delta T}$ has a known covariance matrix $\mathbf{Q}_{\Delta T}$. Rearranging (1), we obtain

$$\begin{aligned}\mathbf{x}_{k-1} &= \mathbf{F}_{\Delta T}^{-1}\mathbf{x}_k - \mathbf{F}_{\Delta T}^{-1}\mathbf{v}_{\Delta T} \\ &= \mathbf{F}_{\Delta T}^{(b)}\mathbf{x}_k + \mathbf{v}_{\Delta T}^{(b)}\end{aligned} \tag{2}$$

where $\mathbf{F}_{\Delta T}^{(b)} = \mathbf{F}_{\Delta T}^{-1}$ is the backward state-propagation matrix, and the backward plant noise sequence $\mathbf{v}_{\Delta T}^{(b)}$ is a zero-mean white Gaussian sequence with covariance matrix

$$\begin{aligned}\mathbf{Q}_{\Delta T}^{(b)} &= \mathbf{F}_{\Delta T}^{-1}\mathbf{Q}_{\Delta T}\mathbf{F}_{\Delta T}^{-\mathrm{T}} \\ &= \mathbf{F}_{\Delta T}^{(b)}\mathbf{Q}_{\Delta T}\left\{\mathbf{F}_{\Delta T}^{(b)}\right\}^{\mathrm{T}}\end{aligned} \tag{3}$$

where T denotes the matrix transpose $\mathbf{F}^{-\mathrm{T}} \equiv \left(\mathbf{F}^{-1}\right)^{\mathrm{T}}$.

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If (2) is extended for N-step-backward propagation, we have

$$\begin{aligned}\mathbf{x}_{k-N} &= \mathbf{F}_{N\Delta T}^{(b)}\mathbf{x}_k - \mathbf{F}_{N\Delta T}^{(b)}\mathbf{v}_{N\Delta T} \\ &= \mathbf{F}_{N\Delta T}^{(b)}\mathbf{x}_k + \mathbf{v}_{N\Delta T}^{(b)}\end{aligned} \tag{4}$$

where $\mathbf{v}_{N\Delta T}^{(b)}$ is a zero-mean white Gaussian plant noise sequence with covariance matrix

$$\mathbf{Q}_{N\Delta T}^{(b)} = \mathbf{F}_{N\Delta T}^{(b)}\mathbf{Q}_{N\Delta T}\left\{\mathbf{F}_{N\Delta T}^{(b)}\right\}^{\mathrm{T}} \tag{5}$$

where $\mathbf{Q}_{N\Delta T}$ is the covariance matrix of the plant noise $\mathbf{v}_{N\Delta T}$.

For fast computation, employing (4) and (5) for one-step-backward propagations is more useful than applying (2) N times in the N-scan window, such that

$$\mathbf{x}_{k-N} = \left(\mathbf{F}_{\Delta T}^{(b)}\right)^N \mathbf{x}_k - \sum_{i=1}^{N}\left(\mathbf{F}_{\Delta T}^{(b)}\right)^i \mathbf{v}_{(N-i+1)\Delta T} \tag{6}$$

where the plant noise covariance matrix can be expressed as

$$\begin{aligned}\mathbf{Q}_{N\Delta T}^{(b)} &= \sum_{i=1}^{N}\left(\mathbf{F}_{\Delta T}^{(b)}\right)^i \mathbf{Q}_{\Delta T}\left\{\left(\mathbf{F}_{\Delta T}^{(b)}\right)^i\right\}^{\mathrm{T}} \\ &= \sum_{i=1}^{N}\left(\mathbf{F}_{\Delta T}^{(b)}\right)^{i-1} \mathbf{Q}_{\Delta T}^{(b)}\left\{\left(\mathbf{F}_{\Delta T}^{(b)}\right)^{i-1}\right\}^{\mathrm{T}}\end{aligned} \tag{7}$$

A special case is needed to choose the form of $\mathbf{Q}_{N\Delta T}$ because (5) should be equivalent to (7).

B. *Sensor*

The target measurement is present at scan k with a probability of detection $\mathrm{P_D}$ and equals

$$\mathbf{Z}_k = \mathbf{H}\mathbf{x}_k + \boldsymbol{\omega}_k \tag{8}$$

where $\mathbf{H}$ is the linear measurement matrix and $\boldsymbol{\omega}_k$ is a zero-mean white Gaussian sequence with known covariance $\mathbf{R}_k$, uncorrelated with the sequence $\mathbf{v}_{\Delta T}$. At each scan, the sensor also returns a random number of clutter measurements, which follow a homogeneous Poisson distribution. The clutter measurement density $\rho(\mathbf{Z})$ is a function of the surveillance space coordinate $\mathbf{Z}$ and is assumed known. The target measurement is validated with probability $\mathrm{P_G}$, and the target is detected with probability $\mathrm{P_D}$.

III. Smoothing

A. *Basic Concept of Smoothing*

Assume that a linear system with Gaussian prior is described by (1) and (8). Denote with $\{\Phi\}$ a sequence of measurement times, and with $\mathbf{Z}^{\{\Phi\}}$ the sequence of corresponding measurement sets. Given no data association issues, the pdf of state estimate $\mathbf{x}_k$ based on $\mathbf{Z}^{\{\Phi\}}$ is

$$p(\mathbf{x}_k \mid \mathbf{Z}^{\{\Phi\}}) = N(\mathbf{x}_k; \hat{\mathbf{x}}_{k|\{\Phi\}}, \mathbf{P}_{k|\{\Phi\}}) \tag{9}$$

where $N(\mathbf{x};\mathbf{m},\boldsymbol{\Sigma})$ is the normal distribution of $\mathbf{x}$, with mean $\mathbf{m}$ and covariance $\boldsymbol{\Sigma}$. Replacing $\{\Phi\}$ with $1:k$, $1:k-1$, $k+1:N$, and $1:N$ (9) indicates the forward estimation pdf, forward prediction pdf, backward prediction pdf, and smoothed estimation pdf, respectively. Here, N is the last scan of the smoothing window with size $N_s = N-k+1$.

Define the fuse operation, which fuses independent estimates defined by their means $\mathbf{m}_1$ and $\mathbf{m}_2$ and their covariances $\mathbf{P}_1$ and $\mathbf{P}_2$ (assuming infinite prior state covariance):

$$[\mathbf{m},\mathbf{P}] = \mathrm{Fuse}(\mathbf{m}_1,\mathbf{P}_1,\mathbf{m}_2,\mathbf{P}_2) \tag{10}$$

by

$$\begin{aligned}\mathbf{P}^{-1} &= \mathbf{P}_1^{-1} + \mathbf{P}_2^{-1} \\ \mathbf{m} &= \mathbf{P}\left(\mathbf{P}_1^{-1}\mathbf{m}_1 + \mathbf{P}_2^{-1}\mathbf{m}_2\right)\end{aligned} \tag{11}$$

Assume that we have forward estimates and backward predictions for scan k with smoothing window:

$$p(\mathbf{x}_k \mid \mathbf{Z}^k) = N(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k}) \tag{12}$$

$$p(\mathbf{x}_k \mid \mathbf{Z}^{k+1:N}) = N(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k+1:N}, \mathbf{P}_{k|k+1:N}) \tag{13}$$

Smoothing estimates can be obtained using fusing forward estimates and backward predictions.

$$\left[\hat{\mathbf{x}}_{k|1:N}, \mathbf{P}_{k|1:N}\right] = \mathrm{Fuse}\left(\hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k}, \hat{\mathbf{x}}_{k|k+1:N}, \mathbf{P}_{k|k+1:N}\right) \tag{14}$$

B. *Retrodictions with Information Filter*

For calculation of smoothed estimates at scan k, both the forward estimated state vector $\hat{\mathbf{x}}_{k|k}$ and its covariance matrix $\mathbf{P}_{k|k}$ and the backward propagated state vector $\hat{\mathbf{x}}_{k|k+1:N}$ and its covariance matrix $\mathbf{P}_{k|k+1:N}$ are required within the smoothing window, as shown in (14). Assume availability of a forward estimator applied to scans $l = 1,\ldots,k$, we obtain forward estimates $\hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k}$. For retrodictions $\hat{\mathbf{x}}_{k|k+1:N}, \mathbf{P}_{k|k+1:N}$, we use the information filter. The information filter recursively backward propagates and updates the information filter state, which for scan l with smoothing window $l = k,\ldots,N$, consists of the information state $\hat{\mathbf{y}}_{l|l+1:N}$ and the inverse of backward prediction error covariance $\mathbf{Y}_{l|l+1:N}$, as follows:

$$\begin{aligned}\mathbf{Y}_{l|l+1:N} &= \mathbf{P}_{l|l+1:N}^{-1} \\ \hat{\mathbf{y}}_{l|l+1:N} &= \mathbf{Y}_{l|l+1:N}\hat{\mathbf{x}}_{l|l+1:N}\end{aligned} \tag{15}$$

In the backward filtering loop, the information filter update function using the measurement $\mathbf{Z}_l$ is presented as follows:

$$\left[\mathbf{Y}_{l|l:N}, \hat{\mathbf{y}}_{l|l:N}\right] = \mathrm{bIF_U}\left[\mathbf{Y}_{l|l+1:N}, \hat{\mathbf{y}}_{l|l+1:N}, \mathbf{Z}_l, \mathbf{R}, \mathbf{H}\right] \tag{16}$$

and defined by

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$$\mathbf{Y}_{l|l:N} = \mathbf{Y}_{l|l+1:N} + \mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H} \tag{17}$$

$$\hat{\mathbf{y}}_{l|l:N} = \hat{\mathbf{y}}_{l|l+1:N} + \mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Z}_l \tag{18}$$

where the initial values of $\mathbf{Y}_{N|N+1:N}$ and $\hat{\mathbf{y}}_{N|N+1:N}$ are a zero matrix and a zero vector, respectively. The information filter update in the backward direction is done, followed by information filter retrodiction with time interval ΔT between scans l and l - 1.

$$\left[\mathbf{Y}_{l-1|l:N}, \hat{\mathbf{y}}_{l-1|l:N}\right] = \mathrm{bIF_P}\left[\mathbf{Y}_{l|l:N}, \hat{\mathbf{y}}_{l|l:N}, \mathbf{F}_{\Delta T}^{(\mathrm{b})}, \mathbf{Q}_{\Delta T}^{(\mathrm{b})}\right] \tag{19}$$

and defined by

$$\mathbf{A}_{l-1} = \left(\mathbf{F}_{\Delta T}^{-(b)}\right)^{\mathrm{T}} \mathbf{Y}_{l|l:N}\mathbf{F}_{\Delta T}^{-(b)} \tag{20}$$

$$\mathbf{Y}_{l-1|l:N} = \left(\mathbf{A}_{l-1}\mathbf{Q}_{\Delta T}^{(b)} + \mathbf{I}\right)^{-1}\mathbf{A}_{l-1} \tag{21}$$

$$\hat{\mathbf{y}}_{l-1|l:N} = \left(\mathbf{A}_{l-1}\mathbf{Q}_{\Delta T}^{(b)} + \mathbf{I}\right)^{-1}\left(\mathbf{F}_{\Delta T}^{-(b)}\right)^{\mathrm{T}}\hat{\mathbf{y}}_{l|l:N} \tag{22}$$

where

$$\mathbf{F}^{-(b)} \equiv \left(\mathbf{F}^{(\mathrm{b})}\right)^{-1} \tag{23}$$

It is straightforward to show that, given the invertibility of $\mathbf{Y}$, the information filter propagation and update are algebraically identical to the Kalman filter propagation and update, respectively.

It is instructive to write the information filter output as a combination of contributions of individual measurements. Employing an induction method using (16) and (19) consecutively, we can derive the following formulas for backward predictions:

$$\mathbf{Y}_{k|k+1:N} = \sum_{l=N}^{k+1}\left(\prod_{s=k}^{l-1}\left(\left(\mathbf{A}_s\mathbf{Q}_{\Delta T}^{(b)} + \mathbf{I}\right)^{-1}\left(\mathbf{F}_{\Delta T}^{-(b)}\right)^{\mathrm{T}}\right)\mathbf{Y}_{l,l}\mathbf{F}_{(l-k)\cdot\Delta T}^{-(b)}\right) \tag{24}$$

$$\hat{\mathbf{y}}_{k|k+1:N} = \sum_{l=N}^{k+1}\left(\prod_{s=k}^{l-1}\left(\left(\mathbf{A}_s\mathbf{Q}_{\Delta T}^{(b)} + \mathbf{I}\right)^{-1}\left(\mathbf{F}_{\Delta T}^{-(b)}\right)^{\mathrm{T}}\right)\hat{\mathbf{y}}_{l,l}\right) \tag{25}$$

where $\mathbf{A}_s$ satisfying (20) is calculated by using $\mathbf{Y}_{s+1|s+1:N}$, and $\mathbf{Y}_{l,l}$ and $\hat{\mathbf{y}}_{l,l}$ are defined as

$$\mathbf{Y}_{l,l} = \mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H} \tag{26}$$

$$\hat{\mathbf{y}}_{l,l} = \mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Z}_l \tag{27}$$

If $\mathbf{Q}_{\Delta T}^{(b)} = \mathbf{0}$, both (24) and (25) can be expressed as a linear combination of the retrodicted information filter states of individual measurements.

$$\mathbf{Y}_{k|k+1:N} = \sum_{l=N}^{k+1}\left(\left(\mathbf{F}_{(l-k)\Delta T}^{-(b)}\right)^{\mathrm{T}}\mathbf{Y}_{l,l}\mathbf{F}_{(l-k)\Delta T}^{-(b)}\right) \tag{28}$$

$$\hat{\mathbf{y}}_{k|k+1:N} = \sum_{l=N}^{k+1}\left(\left(\mathbf{F}_{(l-k)\Delta T}^{-(b)}\right)^{\mathrm{T}}\hat{\mathbf{y}}_{l,l}\right) \tag{29}$$

But if $\mathbf{Q}_{\Delta T}^{(b)} \neq \mathbf{0}$, the information filter state can no longer be represented as a linear combination form like (28) and (29), so for the current scan k, future information $\mathbf{Y}_{l,l}$, $l = k+1,\ldots,L$ is required to calculate the effects of a particular measurement $\mathbf{Z}_L$, $k < L \leq N$, on $\mathbf{Y}_{k|k+1:L}$ and $\hat{\mathbf{y}}_{k|k+1:L}$. If we assume the effects of other information are negligible for calculating the effect of the measurement $\mathbf{z}_L$ on $\mathbf{Y}_{k|k+1:N}$ and $\hat{\mathbf{y}}_{k|k+1:N}$, we can express each of them as a linear combination of contributions of individual measurements. For the window consisting of scans $l = k,\ldots,N$,

$$\mathbf{Y}_{k|k+1:N} \approx \sum_{l=N}^{k+1}\left(\left(\tilde{\mathbf{A}}_{k,l}\mathbf{Q}_{(l-k)\Delta T}^{(b)} + \mathbf{I}\right)^{-1}\tilde{\mathbf{A}}_{k,l}\right) \tag{30}$$

$$\hat{\mathbf{y}}_{k|k+1:N} \approx \sum_{l=N}^{k+1}\left(\left(\tilde{\mathbf{A}}_{k,l}\mathbf{Q}_{(l-k)\Delta T}^{(b)} + \mathbf{I}\right)^{-1}\mathbf{F}_{(l-k)\Delta T}^{-(b)^{\mathrm{T}}}\hat{\mathbf{y}}_{l,l}\right) \tag{31}$$

where $\mathbf{Q}_{(l-k)\Delta T}^{(b)}$ satisfies (5) and (7), and

$$\tilde{\mathbf{A}}_{k,l} = \left(\mathbf{F}_{(l-k)\Delta T}^{-(b)}\right)^{\mathrm{T}}\mathbf{Y}_{l,l}\mathbf{F}_{(l-k)\Delta T}^{-(b)} \tag{32}$$

By using (14), (30), and (31), despite the loss of exactness, we can extend the information filter for data association for smoothing and FTD in cluttered environments.

IV. sIPDA-IR Algorithm for Cluttered Environments

Smoothing estimates is obtained by fusing the forward estimates and backward predictions as shown in (14). Without data association issues, the forward estimates are obtained using standard estimator such as the Kalman filter [2] and backward predictions using (24) and (25) or (30) and (31). In cluttered environments, however, data association is necessary because the origin of the each measurement is uncertain and target measurement is detected with a certain probability of detection.

sIPDA-IR which is the fixed lag smoother calculates the smoothed estimates and the PTE by fusing the forward estimates obtained by forward IPDA [6] and information retrodictions at each scan within the smoothing window iteratively. IPDA formulae is omitted and only smoothing update is concentrated in this section. Let $\hat{\mathbf{x}}_{k|1:l}$, $\mathbf{P}_{k|1:l}$ and $\hat{\mu}_{k|1:l}$ denote smoothed state, covariance and PTE at scan k, respectively (e.g. forward update estimates for $l = k$, smoothed estimates for $k < l \leq N$ within the smoothing window $l = k,\ldots,N$). As scan l close to scan N, the estimation error is decreased due to the smoothing effect.

In order to fuse one forward IPDA estimates and a number of information retrodictions whose origins are unknown, data association is required to discriminate target-originated information from the clutter measurement information. For $l = k+1,\ldots,N$, measurement set $\mathbf{z}_l$ at scan l consists of m_l number of measurement. Information

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retrodictions from scan l to scan k of the i-th measurement $\mathbf{Z}_{l,i} \in \mathbf{z}_l$ obtained at scan l are expressed as follows:

$$\mathbf{Y}_k^{i*}(l) = \left(\tilde{\mathbf{A}}_{k,l}\mathbf{Q}_{(l-k)\cdot\Delta T}^{(b)} + \mathbf{I}\right)^{-1}\tilde{\mathbf{A}}_{k,l} \tag{33}$$

$$\hat{\mathbf{y}}_k^{i*}(l) = \left(\tilde{\mathbf{A}}_{k,l}\mathbf{Q}_{(l-k)\cdot\Delta T}^{(b)} + \mathbf{I}\right)^{-1}\mathbf{F}_{(l-k)\cdot\Delta T}^{-(b)\mathrm{T}}\hat{\mathbf{y}}_{l,l}^{i} \tag{34}$$

where $\hat{\mathbf{y}}_{l,l}^{i} = \mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Z}_{l,i}$.

A. *Transformation of Information Retrodictions*

The measurement likelihood of the retrodicted information state $\hat{\mathbf{y}}_k^{i*}(l)$ is expressed as

$$p_k^{i*}(l) = N\left(\mathbf{Z}_k^{i*}(l)\ ;\hat{\mathbf{x}}_{k|1:l-1}, \mathbf{P}_{k|1:l-1} + \mathbf{P}_k^{i*}(l)\right)/P_G \tag{35}$$

where P_G is the gating probability and

$$\mathbf{P}_k^{i*}(l) = \left(\mathbf{Y}_k^{i*}(l)\right)^{-1}. \tag{36}$$

The equivalent measurement $\mathbf{Z}_k^{i*}(l)$ is defined as

$$\mathbf{Z}_k^{i*}(l) = \left(\mathbf{Y}_k^{i*}(l)\right)^{-1}\hat{\mathbf{y}}_k^{i*}(l) \tag{37}$$

if $\mathbf{Y}_k^{i*}(l)$ is invertible. However, we cannot achieve matrix inversion because the rank of $\mathbf{Y}_k^{i*}(l)$ is less than the state dimension n.

By applying the spectral matrix decomposition theory [9], the retrodicted information $\mathbf{Y}_k^{i*}(l)$ in (33) can be expressed as follows:

$$\mathbf{Y}_k^{i*}(l) = \left(\mathbf{H}^{\mathrm{W}}\right)^{\mathrm{T}}\left(\mathbf{W}_k^{i*}(l)\right)^{-1}\mathbf{H}^{\mathrm{W}} \tag{38}$$

where

$$\mathbf{H}^{\mathrm{W}} = \left[\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p\right]^{\mathrm{T}} \tag{39}$$

$$\left(\mathbf{W}_k^{i*}(l)\right)^{-1} = \mathrm{diag}\left[\lambda_1, \lambda_2, \ldots, \lambda_p\right] \tag{40}$$

Here, "diag" means a diagonal matrix. $\left[\lambda_1, \lambda_2, \ldots, \lambda_p\right]$ are the nonzero eigenvalues of $\mathbf{Y}_k^{i*}(l)$, and $\left[\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p\right]$ are the corresponding orthonormal eigenvectors of $\mathbf{Y}_k^{i*}(l)$. Finally, the invertible part of $\mathbf{Y}_k^{i*}(l)$ can be represented as

$$\mathbf{W}_k^{i*}(l) = \left(\mathbf{H}^{\mathrm{W}}\mathbf{Y}_k^{i*}(l)\left(\mathbf{H}^{\mathrm{W}}\right)^{\mathrm{T}}\right)^{-1} \tag{41}$$

and the projection of the equivalent measurement on the invertible space can be represented as

$$\begin{aligned}\hat{\mathbf{w}}_k^{i*}(l) &= \mathbf{H}^{\mathrm{W}}\mathbf{Z}_k^{i*}(l) \\ &= \mathbf{W}_k^{i*}(l)\left(\mathbf{H}^{\mathrm{W}}\hat{\mathbf{y}}_k^{i*}(l)\right)\end{aligned} \tag{42}$$

The projected equivalent measurement $\hat{\mathbf{w}}_k^{i*}(l)$ is used to form the measurement likelihood in (35) for data association.

B. *Smoothing Innovations for Data Association*

The smoothing update starts with the forward update estimates $\hat{\mathbf{x}}_{k|k}$, $\mathbf{P}_{k|k}$ and $\hat{\mu}_{k|k}$. Let $\Upsilon_k(l)$ be the set of information retrodictions in (33) and (34) :

$$\Upsilon_k(l) = \left[\left\{\hat{\mathbf{y}}_k^{1*}(l), \mathbf{Y}_k^{1*}(l)\right\}, \ldots, \left\{\hat{\mathbf{y}}_k^{m_l*}(l), \mathbf{Y}_k^{m_l*}(l)\right\}\right] \tag{43}$$

Smoothing estimates is obtained by fusing the estimates $\hat{\mathbf{x}}_{k|1:l-1}, \mathbf{P}_{k|1:l-1}$ and the set of information retrodictions $\Upsilon_k(l)$, iteratively for $l = k+1, \ldots, N$:

$$\begin{aligned}&\left[\hat{\mathbf{x}}_{k|1:l}, \mathbf{P}_{k|1:l}, \hat{\mu}_{k|1:l}\right] \\ &\quad = \text{sIPDA-IR}_{\mathrm{U}}\left[\hat{\mathbf{x}}_{k|1:l-1}, \mathbf{P}_{k|1:l-1}, \hat{\mu}_{k|1:l-1}, \Upsilon_k(l)\right]\end{aligned} \tag{44}$$

Before calculating the likelihood pdf of the information retrodiction state $\hat{\mathbf{y}}_k^{i*}(l)$, $\hat{\mathbf{y}}_k^{i*}(l)$ should be expressed in W space using (41) and (42) because its inverse covariance $\mathbf{Y}_k^{i*}(l)$ is singular. Then, the measurement likelihood becomes

$$p_k^{i*}(l) = N\left(\hat{\mathbf{w}}_k^{i*}(l); \hat{\mathbf{x}}_{k|1:l-1}^{\mathrm{W}}, \mathbf{P}_{k|1:l-1}^{\mathrm{W}} + \mathbf{W}_k^{i*}(l)\right)/P_G \tag{45}$$

where the projected estimates are expressed as

$$\hat{\mathbf{x}}_{k|1:l-1}^{\mathrm{W}} = \mathbf{H}^{\mathrm{W}}\hat{\mathbf{x}}_{k|1:l-1} \tag{46}$$

$$\mathbf{P}_{k|1:l-1}^{\mathrm{W}} = \mathbf{H}^{\mathrm{W}}\mathbf{P}_{k|1:l-1}\left(\mathbf{H}^{\mathrm{W}}\right)^{\mathrm{T}} \tag{47}$$

As shown in (46) and (47), smoothed state $\hat{\mathbf{x}}_{k|1:l-1}$ and its covariance $\mathbf{P}_{k|1:l-1}$ are also expressed in the invertible space using the projection matrix $\mathbf{H}^{\mathrm{W}}$.

The measurement likelihood ratio of the retrodicted equivalent measurement set $\hat{\mathbf{w}}_k^{*}(l) = \left\{\hat{\mathbf{w}}_k^{i*}(l),\ i = 1, \ldots, m_l\right\}$ at scan k satisfies

$$\begin{aligned}\Lambda_k^{*}(l) &\triangleq \frac{p\left(\hat{\mathbf{w}}_k^{*}(l), m_l \mid \chi_k, \mathbf{Z}^{l-1}\right)}{p\left(\hat{\mathbf{w}}_k^{*}(l), m_l \mid \bar{\chi}_k, \mathbf{Z}^{l-1}\right)} \\ &= 1 - \mathrm{P_D P_G} + \mathrm{P_D P_G}\sum_{i=1}^{m_l}\frac{p_k^{i*}(l)}{\rho_k^{i*}(l)}\end{aligned} \tag{48}$$

where $\chi_k, \bar{\chi}_k$ denote target existence and nonexistence, respectively. Finally, the smoothed probability of target existence $\hat{\mu}_{k|1:l} \equiv P\left(\chi_k \mid \mathbf{Z}^l\right)$ is

$$\hat{\mu}_{k|1:l} = \frac{\Lambda_k^{*}(l)\hat{\mu}_{k|1:l-1}}{1 - \left(1 - \Lambda_k^{*}(l)\right)\hat{\mu}_{k|1:l-1}} \tag{49}$$

Denote with $\theta_{k,i}(l); i \geq 0$ the event that retrodicted information state $\hat{\mathbf{y}}_k^{i*}(l)$ is target-originated. Non-detection implies i = 0. The data association probability for $\theta_{k,i}(l)$ is defined as

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$$\beta_{k,i}^{*}(l)=P\left(\theta_{k,i}(l)\mid\chi_k,\mathbf{Z}^l\right) \tag{50}$$

Using Bayes rule, the data association probability becomes [3]

$$\beta_{k,i}^{*}(l)=\frac{1}{\Lambda_k^{*}(l)}\begin{cases}1-\mathrm{P_D P_G}, & i=0\\ \mathrm{P_D P_G}\,\dfrac{p_k^{i*}(l)}{\rho_k^{i*}(l)}, & i>0\end{cases} \tag{51}$$

C. *Update of Smoothed Estimates*

Since the events $\theta_{k,i}(l); i\geq 0$ are mutually exclusive and exhaustive, using the total probability theorem results in the following *a posteriori* pdf for the target state:

$$p\left(\mathbf{x}_k\mid\chi_k,\mathbf{Z}^l\right)=\sum_{i=0}^{m_l}\beta_{k,i}^{*}(l)\,p\left(\mathbf{x}_k\mid\theta_{k,i}(l),\chi_k,\mathbf{Z}^l\right) \tag{52}$$

where

$$p\left(\mathbf{x}_k\mid\theta_{k,0}(l),\chi_k,\mathbf{Z}^l\right)=p\left(\mathbf{x}_k\mid\chi_k,\mathbf{Z}^{l-1}\right) \tag{53}$$

$$\begin{aligned}p\left(\mathbf{x}_k\mid\theta_{k,i>0}(l),\chi_k,\mathbf{Z}^l\right)&=p\left(\mathbf{x}_k\mid\hat{\mathbf{w}}_k^{i*}(l),\chi_k,\mathbf{Z}^{l-1}\right)\\&=\frac{p\left(\hat{\mathbf{w}}_k^{i*}(l)\mid\mathbf{x}_k,\chi_k,\mathbf{Z}^{l-1}\right)}{p\left(\hat{\mathbf{w}}_k^{i*}(l)\mid\chi_k,\mathbf{Z}^{l-1}\right)}\,p\left(\mathbf{x}_k\mid\chi_k,\mathbf{Z}^{l-1}\right)\end{aligned} \tag{54}$$

All pdfs in (53), (54) are approximated by Gaussian pdfs:

$$p\left(\mathbf{x}_k\mid\theta_{k,i}(l),\chi_k,\mathbf{Z}^l\right)\approx N\left(\mathbf{x}_k;\hat{\mathbf{x}}_{k|1:l}^{i},\mathbf{P}_{k|1:l}^{i}\right) \tag{55}$$

where for i = 0,

$$\left[\hat{\mathbf{x}}_{k|1:l}^{0},\mathbf{P}_{k|1:l}^{0}\right]=\left[\hat{\mathbf{x}}_{k|1:l-1},\mathbf{P}_{k|1:l-1}\right] \tag{56}$$

and for i > 0,

$$\left[\hat{\mathbf{y}}_{k|1:l}^{i},\mathbf{Y}_{k|1:l}^{i}\right]=\mathrm{IF_U}\left[\hat{\mathbf{y}}_{k|1:l-1},\mathbf{Y}_{k|1:l-1},\hat{\mathbf{y}}_k^{i*}(l),\mathbf{Y}_k^{i*}(l)\right] \tag{57}$$

where

$$\mathbf{Y}_{k|1:l}^{i}=\mathbf{Y}_{k|1:l-1}+\mathbf{Y}_k^{i*}(l) \tag{58}$$

$$\hat{\mathbf{y}}_{k|1:l}^{i}=\hat{\mathbf{y}}_{k|1:l-1}+\hat{\mathbf{y}}_k^{i*}(l) \tag{59}$$

Here, $\mathrm{IF_U}$ denotes the standard information filter update.

$\mathbf{Y}_{k|1:l-1}$ and $\hat{\mathbf{y}}_{k|1:l-1}$ are obtained from $\mathbf{P}_{k|1:l-1}$ and $\hat{\mathbf{x}}_{k|1:l-1}$ using (15):

$$\mathbf{Y}_{k|1:l-1}=\mathbf{P}_{k|1:l-1}^{-1} \tag{60}$$

$$\hat{\mathbf{y}}_{k|1:l-1}=\mathbf{Y}_{k|1:l-1}\hat{\mathbf{x}}_{k|1:l-1} \tag{61}$$

where $\mathbf{P}_{k|1:l-1}$ for $l\geq k+1$ is invertible because $\mathbf{P}_{k|k}$ is non-singular. After the information update of (57), one can obtain $\mathbf{P}_{k|1:l}^{i}$ and $\hat{\mathbf{x}}_{k|1:l}^{i}$ from $\mathbf{Y}_{k|1:l}^{i}$ and $\hat{\mathbf{y}}_{k|1:l}^{i}$. Then, the Gaussian mixture defined in (52) is approximated with a single Gaussian pdf with mean $\hat{\mathbf{x}}_{k|1:l}$ and covariance $\mathbf{P}_{k|1:l}$:

$$p\left(\mathbf{x}_k\mid\chi_k,\mathbf{Z}^l\right)\approx N\left(\mathbf{x}_k;\hat{\mathbf{x}}_{k|1:l},\mathbf{P}_{k|1:l}\right) \tag{62}$$

where

$$\hat{\mathbf{x}}_{k|1:l}=\sum_{i=0}^{m_l}\beta_{k,i}^{*}(l)\hat{\mathbf{x}}_{k|1:l}^{i} \tag{63}$$

$$\mathbf{P}_{k|1:l}=\sum_{i=0}^{m_l}\beta_{k,i}^{*}(l)\left(\mathbf{P}_{k|1:l}^{i}+\hat{\mathbf{x}}_{k|1:l}^{i}\hat{\mathbf{x}}_{k|1:l}^{i\,\mathrm{T}}\right)-\hat{\mathbf{x}}_{k|1:l}\hat{\mathbf{x}}_{k|1:l}^{\mathrm{T}} \tag{64}$$

Calculation of (43)-(64) is continued iteratively until $l=N$.

Note that the forward process of the sIPDA-IR algorithm is operated independently from the smoothed variables. The smoothed estimates are only used for filter output, and they are not accumulated for the next smoothing window. Therefore, the smoothed estimates obtained at each scan do not affect the smoothed estimates of other scans. For this reason, there is no data incest problem in the proposed algorithm.

V. Simulation Study

We consider a two-dimensional surveillance scenario. The surveillance area is 600 m long (x-axis) and 400 m wide (y-axis). The trajectory state consists of the two-dimensional position and the velocity vectors. The single target exists with the initial trajectory state of $\mathbf{x}_0=\left[50\mathrm{m}\ 200\mathrm{m}\ 15\mathrm{m/s}\ 0\mathrm{m/s}\right]^{\mathrm{T}}$ and then propagates according to (1), with

$$\mathbf{F}_{\Delta T}=\begin{bmatrix}\mathbf{I}_2 & \Delta T\mathbf{I}_2\\ \mathbf{0}_{2,2} & \mathbf{I}_2\end{bmatrix},\ \mathbf{Q}_{\Delta T}=q\begin{bmatrix}\frac{\Delta T^3}{3}\mathbf{I}_2 & \frac{\Delta T^2}{2}\mathbf{I}_2\\ \frac{\Delta T^2}{2}\mathbf{I}_2 & \Delta T\mathbf{I}_2\end{bmatrix} \tag{65}$$

where $\Delta T=1\mathrm{s}$ denotes the measurement (scan) interval, $\mathbf{I}_n$ denotes the n-dimensional identity matrix, $\mathbf{O}_{m,n}$ denotes the m-by-n zero matrix, and q is 0.1 m^2/s^3.

A linear sensor provides the target position measurements with detection probability 0.9 and measurement covariance $\mathbf{R}=25\mathbf{I}_2 m^2$. The clutter measurement density is 10^{-4} m^2. Each simulation run consists of 36 scans, and each experiment averages the outcomes from 1000 runs. New tracks are initialized every scan using all possible measurement pairs in consecutive scans (two-point differencing) [3] with the maximum velocity constraint, 25 m/s. Roughly 150,000 (4.2 per scan) new tracks are initialized per experiment.

We compare the IPDA, the sIPDA, and the sIPDA-IR algorithms. The smoothing algorithms sIPDA and sIPDA-IR are implemented using a 5-scan smoothing window. For example, when we calculate smoothed estimates at scan 2, the range of the smoothing window is from scan 2 to scan 6.

In the smoothing algorithms, tracks are terminated if either forward update PTE or smoothed update PTE falls below a predetermined termination threshold. Tracks are confirmed if the smoothed update PTE exceeds a predetermined confirmation threshold. IPDA uses the updated PTE to terminate and confirm tracks. The FTD parameters are tuned to deliver □ 30 confirmed false tracks for each algorithm. Figures 1 and 2 show the confirmed true track (CTT) rate and root mean square (RMS) errors of CTT following a target. The CTT rate increases toward 100%,

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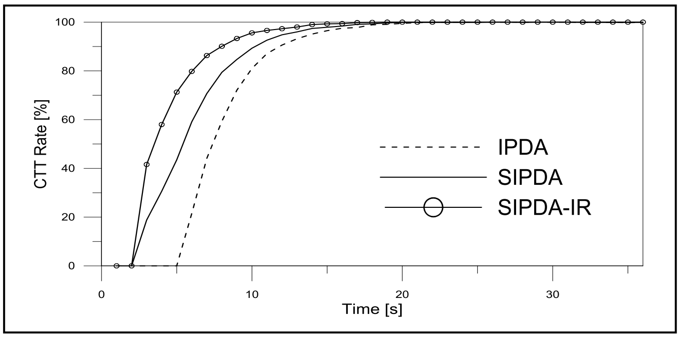

Figure 1. Confirmed true track rates

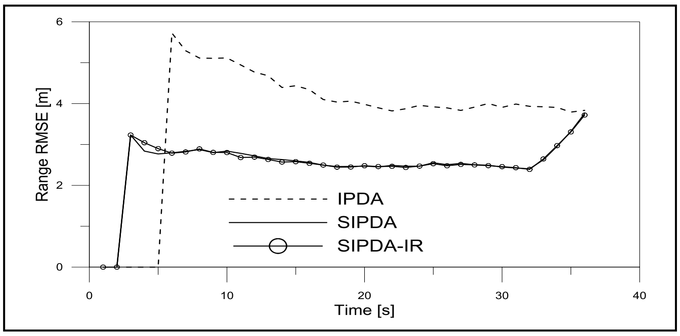

Figure 2. Root mean square errors

and among them, sIPDA-IR shows the fastest response. Because the sIPDA algorithm is an RTS-based smoother, the smoothed estimates can be obtained if the track is retained until scan *N* (which is the last scan of the smoothing window). If not, sIPDA does not have a chance to calculate the smoothed estimates. However, our proposed algorithm does not need updated estimates from k+1 to N for smoothing updates, but it needs retrodictions of information gathered in the window to scan k. This results in a fast increase of the CTT rate. The RMS errors of the proposed algorithm are less than those of IPDA and similar to sIPDA. But the errors may increase for target tracking with high maneuvering indexes due to the approximation made in (30) and (31).

VI. Conclusions

The smoothing data association algorithms of estimation theory have been used to improve FTD capabilities and reduce estimation errors, incurring the expense of deferred estimation. In this paper, we present the fixed-lag smoothing data association algorithm, sIPDA-IR, for target tracking in clutter. The proposed algorithm fuses forward estimates and measurement information retrodictions within the smoothing window, recursively, until the smoothed estimates at the current scan k are obtained. The simulation studies verify that the proposed sIPDA-IR algorithm shows better FTD performance than both sIPDA and IPDA, and similar RMS errors to sIPDA.

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