

# Development of Static Voltage Stability Index for Weak Bus Identification

M. Jalboub  
Faculty of Eng. Azzawia  
University  
Azzawia-Libya

H. Rajamani  
School of Eng. D&T  
Bradford University, UK

Amer A. Amar  
Faculty of Eng. Azzawia  
University  
Azzawia-Libya

Abdalbaset M. Ihbal  
Faculty of Eng. Azzawia  
University  
Azzawia-Libya

**Abstract-** To meet an ever-increasing electrical load demand, modern power systems are undergoing numerous changes and becoming more complex from operation, control and stability maintenance standpoints. The major problem that is associated with such a stressed system is voltage instability or collapse. A system is said to enter a state of voltage instability when a disturbance causes a progressive and uncontrollable decline in voltage, which can occur because of the inability of the network to meet the increased demand for reactive power. Therefore, the voltage stability analysis is essential in order to identify the critical buses in a power system i.e., buses which are close to their voltage stability limits and thus enable certain measures to be taken by the operators in order to avoid any incidence of voltage collapse. Based on the basic understanding of voltage collapse and voltage stability analysis, a new index of analyzing static stability is proposed utilizing the power flow solution based on the transmission line impedance and the apparent power at the receiving end. The proposed line stability index has the capability to identify the weakest bus and the weakest line in the interconnected power. Simulations were carried out to validate the proposed index using Western System Coordinate Council (WSCC) 9-bus test system. The results are presented and compared with indices found in literature.

**Keywords:** Voltage stability, Stability indices, WSCC 9-bus test system

## I. Introduction

Different voltage instability indicators are used to quantify the proximity of a particular operating state to the point of Voltage Collapse. These indicators can classify different power system conditions. A more practical indicator is the distance between actual load and maximum load at the stability margin in the direction of an estimated or forecasted load increase [1-7].

The analysis of these indicators we find the following: In PV and QV curves, the effects of changes in both active and reactive powers on voltage stability cannot be observed simultaneously. In addition, because of the heavy dependence of these methods to system conditions and type of simulation, the possibility of error is high in these methods. The main drawback of  $V/V_0$  Index is that it presents a highly non-linear profile with respect to changes on the system parameters, not allowing for accurate predictions of proximity to voltage collapse. Line index  $FVSI$ ,  $L_{mn}$ ,  $LQP$  and  $L_{VSI}$  are formulated

either from the relationship between reactive power and voltage or from the relationship between the active power and voltage. All these indices may fail under some special conditions. Examples of possible failure are:

Special branch parameters: the transmission line resistance  $r = 0$ , thus the index  $L_{VSI}$  fails.  $L_{VSI}$  may mis-identify a heavy line as a bottleneck when  $(\theta-\delta)$  approaches to  $90^\circ$ . When  $(\theta-\delta)$  approaches  $90^\circ$ ,  $\cos(\theta-\delta)$  approaches zero. As  $\cos(\theta-\delta)$  is in the denominator, there may be a dramatic increase in  $L_{VSI}$ . It implies that a healthy line may be identified as a critical line.  $L_{VSI}$  is more sensitive to  $\delta$  than  $L_{mn}$  because  $\cos(\theta-\delta)$  changes much faster than  $\sin(\theta-\delta)$  around  $90^\circ$ .  $FVSI$  is derived assuming voltage angle difference is zero. It implies that simplified  $FVSI$  is not suitable for heavy load lines due to large angle difference.  $LQP$  directly involves both of active power and reactive power in the calculation while  $FVSI$  and  $L_{mn}$  directly involve only reactive power, the performance of the  $LQP$  index is slightly better than that of the others.

Modal analysis approach [8] involves the computation of a small number of eigenvalues and associated eigenvectors of a reduced Jacobian matrix which retains the Q-V relationships in the network and includes the appropriate characteristics of generators, loads, reactive power compensating devices (FACTS devices), and HVDC converters. By using the reduced Jacobian matrix instead of the system state matrix, the focus is on the voltage and reactive power characteristics. The eigenvalues of the Jacobian identify different modes through which the system could become voltage unstable. The magnitudes of the eigenvalues provide a relative measure of proximity to instability. The eigenvectors, on the other hand, provide information related to the mechanism of loss of voltage stability. A system is voltage stable at a given operating condition if, for every bus in the system, bus voltage magnitude increases as reactive power injection at the same bus is increased. A system is voltage unstable if, for at least one bus in the system, bus voltage magnitude decreases as the reactive power injection at the same bus is increased. In other words, a system is stable if V-Q sensitivity is positive for every bus and unstable if V-Q sensitivity is negative for at least one bus.

Modal analysis method is applied to the reduced Jacobian matrix in order to identify the weakest based on the calculation of the minimum eigenvalue and the highest participating factor corresponding to that eigenvalue which

represent the most critical bus in the test system. The result will be compared to the results obtained by the proposed index.

## II. Proposed Index Formulation

The characteristics of voltage stability are illustrated by a simple two bus power system shown in Figure 1  $V_i$  sending end voltage,  $V_j$  receiving end voltage,  $Z_{ij}$  transmission line impedance,  $S_i$  sending apparent power, and  $S_j$  receiving end apparent power.

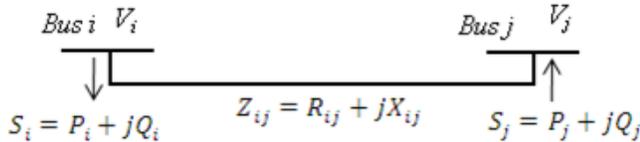


Figure 1 Single line diagram of two bus system

The quadratic equation for the voltage at receiving end is given by:

$$\begin{aligned} &|V_j|^4 + |V_j|^2 \left[ 2(P_j R_{ij} + Q_j X_{ij}) - |V_i|^2 \right] + R_{ij}^2 P_j^2 + X_{ij}^2 Q_j^2 + X_{ij}^2 P_j^2 + R_{ij}^2 Q_j^2 \\ &= 0 \end{aligned}$$

The solution for the receiving end voltage  $V_j$  is:

$$|V_j| = \frac{\sqrt{|V_i|^2 - 2(P_j R_{ij} + Q_j X_{ij})} \pm \sqrt{\left[ |V_i|^2 - 2(P_j R_{ij} + Q_j X_{ij}) \right]^2 - 4(R_{ij}^2 + X_{ij}^2)(P_j^2 + Q_j^2)}}{2}$$

The solution for the receiving end voltage  $V_j$  is given by Equation 3.34. Therefore, in order to simplify Equation 3.34 some simplification must be made. Let:

$$a = |V_i|^2 - 2(P_j R_{ij} + Q_j X_{ij}) \text{ and } b = (R_{ij}^2 + X_{ij}^2)(P_j^2 + Q_j^2)$$

$$\therefore |V_j| = \pm \sqrt{\frac{a \pm \sqrt{a^2 - 4b}}{2}}$$

There are four solutions for  $V_j$ : Two positive and two negative,  $V_j$  must be non-negative, therefore the two negative solution are not true. To derive a positive feasible solution for a real system, the expression under the square root signs must be

positive. Therefore,  $a \pm \sqrt{a^2 - 4b} \leq 1$  or  $a^2 - 4b = 0$ . Since  $b$  is greater than zero, therefore  $a$  must be non-negative for the condition to be satisfied. or  $0 < \frac{2\sqrt{b}}{a} \leq 1$ . By substituting  $a$  and  $b$  we define new voltage stability index is:

$$LSZ = 2 \frac{\sqrt{(R_{ij}^2 + X_{ij}^2)(P_j^2 + Q_j^2)}}{|V_i|^2 - 2(P_j R_{ij} + Q_j X_{ij})}$$

Simplifying:

$$LSZ = 2 \frac{|Z||S|}{|V_i|^2 - 2|Z|(P_j \cos \theta + Q_j \sin \theta)} \leq 1$$

Where  $Z$  is the transmission line impedance,  $S$  is the apparent power  $P_j$  and  $Q_j$  are the active and reactive power at the receiving end,  $\theta$  is the impedance angle and  $V_i$  is the sending end voltage.

The line that exhibits  $LSZ$  closed to 1.00 implies that it is approaching its instability point. If  $LSZ$  goes beyond 1.00, one of the buses to the connected to the line will experience a sudden voltage drop leading to system collapse. The transmission line with the largest value of  $LSZ$  is taken as the weakest line and must receive special care to maintain voltage stability within a certain limit.

In order to investigate the effectiveness of the proposed index, Matlab (Simulation) simulation was Western System Coordinate Council (WSCC) 3-machine, 9-bus test system. In the following section the simulation for WSCC test system is presented and investigated.

## III. Test System Description

To test the proposed index, the voltage stability analysis was conducted with the help of the Matlab program given (Appendix B) on the well-known Western System Coordinate Council (WSCC) 3-machine, 9-bus test system [9] shown in Figure 1. The test system connected in a loop configuration, consists essentially of nine buses (B1 to B9) interconnected through transmission lines (L1, L2, L3), three power plants and three loads connected at buses B5, B6, and B8 respectively, test system data is given in [9].

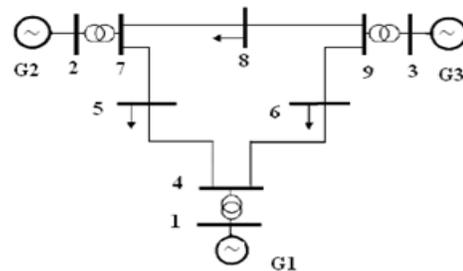


Figure 1 WSCC 3-machines, 9-bus test system [9]

### A. Experiments

To assess the static voltage stability of the WSCC test power system the reactive power loading is increased gradually at a

chosen bus each time until it is near to voltage collapse whilst keeping the loads on the other buses constant. From the results, the proposed index *LSZ* was calculated at each line for every load change. Performing power flow programs on the proposed test system, the following steps been calculated:

- Calculation of *LSZ* by increasing the reactive power at each bus until near to voltage collapse.
- Calculating the voltage profile for all buses.
- Determination of voltage stability margin.

### B. Simulation Results and discussion

The test system described in the previous section is simulated and load flow equations are solved to study the effectiveness of the proposed index and to determine the weakest bus in the WSCC test system. The stability index was calculated for all buses but only buses 5, 6, and 8 are presented here as they are ranked as the weakest buses in the test system. Figure 2 shows the *LSZ* is plotted versus reactive power load change at buses (5, 6, and 8).

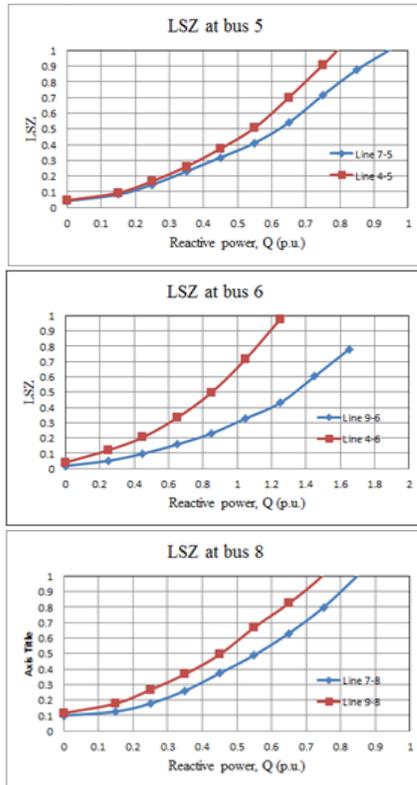


Figure 2 *Lsz* vs reactive power loading at buses (5, 6, 8) of WSCC test system

It is clear from the figures that the line with highest *LSZ* is ranked as the most sensitive line. Therefore, the line connecting bus 4 to bus 5 is the most critical line with respect to bus 5, and the line connecting bus 4 to bus 6 is the most critical line with respect to bus 6. Similarly the line connecting bus 9 to bus 8 is the most critical line with respect to bus 8.

From the results, bus 8 is ranked as the most sensitive bus (weakest bus) because it has the minimum load-ability compared with the other lines.

## IV. Weak Bus Identification by Using Modal Analysis

The modal analysis method is applied to the same test system. The voltage profile of the buses is presented from the load flow simulation. Then, the minimum eigenvalue of the reduced Jacobian matrix is calculated. After that, the weakest load buses, which are subject to voltage collapse, are identified by computing the participating factors. The results are shown in Figure 3.

Figure 3 shows the voltage profile of all buses of the Western System Coordinating Council (WSCC) 3-Machines 9-Bus system as obtained from the load flow program. It can be seen that all the bus voltages are within the acceptable level ( $\pm 10\%$ ). The lowest voltage compared to the other buses can be noticed in bus 8. The load flow result agrees with the proposed index simulated results

Since there are nine buses among which there is one swing bus and two PV buses, then the total number of eigenvalues of the reduced Jacobian matrix  $J_R^{-1}$  is expected to be six as shown in Table 1.

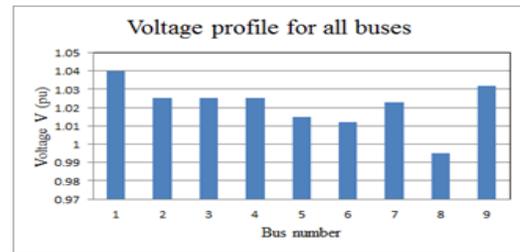


Figure 3 Voltage profiles of all buses of WSCC test system

Table I. WSCC 3-Machines 9-Bus system eigenvalues

#	1	2	3	4	5	6
Eigen value	51.094	46.6306	5.9590	12.9438	14.9108	36.305

Note that from Table 1, all the eigenvalues are positive which means that the system voltage is stable. Also, it can be noticed that the minimum eigenvalue  $\lambda = 5.9590$  is the most critical mode.

The participating factor for this mode has been calculated and the result is shown in Figure 4. The result shows that the buses 5, 6 and 8 have the highest participation factors to the critical mode. The largest participation factor value (0.3) at bus # 8 indicates the highest contribution of this bus to the voltage collapse.





Figure 4 The participating factor of all buses for most critical mode of the test system

## V. Conclusions

In this paper, a new static voltage stability index related to the voltage drop in the critical bus was proposed and investigated. It is referred to as the LSZ index. It is based on the transmission line impedance and the apparent power at the receiving end. The proposed index determines the maximum load that is possible to be connected to a bus in order to maintain the voltage stability margin before the system suffers from voltage collapse. In order to investigate the effectiveness of the proposed index the WSCC 3-machine, 9-bus test system was used. The simulation result detects clearly the stressed condition of the lines, identifies with degree of accuracy the weakest buses prone to voltage collapse and determines the static stability margin of the power to work in save operation. The most effective way for power networks to improve the voltage profile and static voltage stability margin at the weakest bus is by introducing FACTS system. The modal analysis method was applied to the reduced Jacobian matrix in order to identify the weakest based on the calculation of the minimum eigenvalue and the highest participating factor corresponding to that eigenvalue which represent the most critical bus in the test system. The result shows an agreement with the proposed index. The Q-V curve was used to determine the Mvar distance to the voltage instability point. The margins were determined between the base case loading points and the maximum loading points before the voltage collapse. Consequently,

these curves can be used to predict the maximum-stability margins that can be reached. The simulation shows an agreement between the studied line stability indices with the best result obtained by the proposed line stability index LSZ. The computed results show an agreement with other indices found in the literature [10, 11].

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