Auto-Focusing for Mobile Phone and Web Camera

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Abstract --- **Lens defocus causes image blurring. Restoring these kinds of images is an interesting research field. Current auto focus solutions used in commercial cameras are design to ensure that captured images are in focus by adjusting the lens position .A motor is used to move the position of camera lens along the optical axis to take multiple pictures. Disadvantages of auto focus solution are that it requires focal length changing lens and accurate engine to move the lens with a particular step size. Moreover it has fundamental limitations that when the scene contains multiple objects with largely varying depths , a single image cannot capture all the objects in focus simultaneously**

We present a novel method for virtual focus and object depth estimation from defocused video captured by a moving camera. Virtual Focusing stands for the technique of providing an image processing solution to recover focused image sequences from videos taken by an out-of-focus camera with fixed physical parameters (cell phone cameras and webcams).

Keywords — **Focusing, image restoration.**

I. INTRODUCTION

Focusing is an important issue in digital camera design. Current auto-focus solutions widely applied in industry are mostly based on different focus measures. They search for the best focused images while moving the lens and can be tuned to perform fast. The shortage is that they require a focal-length changing lens and an accurate engine that can move the lens with a particular step size. From a different point of view however, image processing solutions model the out-of-focus phenomenon as focused images passing through a linear system. With the estimation of the point spread function (PSF) of the lens system, the focused images can be recovered through a deconvolution process. Discussion in this paper will lie mainly within this class and concentrate on PSF estimation. The overall philosophy of estimating PSF and its Fourier Transform, also known as optical transfer function (OTF), is based on a fundamental observation that is the blur characteristic relates only to the object depth and the camera settings. Despite the fact that the relationship is usually approximated by first order optics, this observation verifies itself through The success of depth-from-defocus (DFD) algorithms. For example, designer in [1] utilizes two settings of camera parameters for acquiring two differently blurred images. Assuming the PSF to be a Gaussian function, a close form solution of the blur parameter is given. More generally, the authors of [2] approximate the underlying OTF by a parametric polynomial and estimate the coefficients using a least-square criteria. In a more recent work [3], the technique of DFD is combined with stereo pairs and the estimation is performed with the tool of Markov random fields to improve the accuracy.

DFD for estimating the PSF has a solid and elegant theoretical foundation however, it poses a high requirement on the hardware. Due to the fact that changing camera settings such as camera aperture and focal length cannot be done without sophisticated experimental device, it limits the applications in practice. The algorithm proposed in this paper, on the other hand, is designed for a 'rigid' camera whose physical parameters are all fixed. Therefore it can be applied to simple digital cameras especially mobile-phone cameras. The other novelty of our algorithm is to exploit multiple images taken by a moving camera. Multiple images taken in variable positions not only provide differently blurred images but also reveal additional resources for improving estimation.

The rest of this paper is organized as follows. In Section 2, we begin with the problem definition and model formulation. In Section 3, we explain the main idea of blur estimation through three examples of PSF. We then discuss in Section 4 the idea of multiple-image estimation and in Section 5 noise analysis for the system. Section 6 provides the simulation results and Section 7 draws the conclusion.

II. CAMERA AND IMAGING MODEL

Assume a moving camera is looking at a static object and taking a video of it. The camera is a rigid camera, meaning that it has a fixed lens aperture, focal length and image plane-to-lens distance. One point in the object projects onto different image coordinates when the camera moves. In time t and time t _, the camera takes two images, frame k and frame k _ . The pixel locations ($x0$, $y0$) in image frame k and ($x1$, $y1$) in frame k _ are related by a 2D affine transform [4]

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \frac{z_0 - f}{z_1 - f} \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \quad (1)$$

where $r11$, $r12$, $r21$, $r22$ are rotation parameters and tx, ty are translation parameters of the camera motion. $z0$ and $z1$ are distances between the object and the camera lens, in time t and time t _ respectively, which are commonly referred as depths of the object. f is the focal length.

Difine s = $\frac{z0-f}{z1-f}$ represents scaling factor Denote the Fourier transform of frame k and frame k _ as $F0(u, v)$ and $F1(u, v)$. According to the affine theorem for 2D Fourier transform [5], the amplitude of $F0(u, v)$ and $F1(u, v)$ has the following relationship:

$$|F_1(u,v)| = \frac{1}{|\Delta|}|F_0(\frac{sr_{22}u - sr_{21}v}{\Delta}, \frac{-sr_{12}u + sr_{11}v}{\Delta})|. \quad (2)$$

Where $\Delta \triangleq s2(z0\text{-}f)/(z1\text{-}f)$. Using the motion estimation and stabilization technique in [4], we can compensate for rotation before we further process the images. Therefore we only discuss here when the rotation matrix is identity. Then (2) can be simplified as

$$|F_1(u,v)| = \frac{1}{s^2}|F_0(\frac{u}{s}, \frac{v}{s})|. \quad (3)$$

When a camera is out of focus, the resulting image is blurred by a specific PSF, whose parameters are uniquely determined by the blur radius R. In frequency domain, the spectrum of blurred image $Y(u, v)$ will be the original spectrum times the OTF $H(u, v,R)$.

$$Y_i(u,v) = F_i(u,v)H(u,v,R_i), \qquad i = 0, 1. \quad (4)$$

Here we consider the PSF being a symmetric (even) function whose Fourier transform is real. With (3) and (4), we have

$$s^2|Y_1(u,v)| = |Y_0(\frac{u}{s}, \frac{v}{s})| \frac{H(u,v,R_1)}{H(u/s, v/s, R_0)}.$$

To proceed, we will incorporate the knowledge from optic geometry. As illustrated in [1], the blur radiuses are given by

$$R_i = vL(\frac{1}{f} - \frac{1}{z_i} - \frac{1}{v}), \qquad i = 0, 1. \quad (6)$$

where v is the image plane-to-lens distance, L is the radius of lens aperture, and z is the depth of the object. It can be seen that the blur radius is affected only by the depth of the object for one particular camera. From the definition of s we can continue to arrive at

$$s = \frac{z_0 - f}{z_1 - f} = \frac{R_0 + L}{R_1 + L} \times \frac{R_1 + L - vL/f}{R_0 + L - vL/f}. \quad (7)$$

To estimate the focused images from the blurred images, we need to estimate the OTF, which equals identifying the blur radiuses. With v,L, f being known camera parameters, it is able to solve for s,$R0$,$R1$, thus$H(u, v,R0)$ and$H(u, v,R1)$, based on (5) and (7).

III. BLUR ESTIMATION AND IMAGE RECONSTRUCTION

In this section, we will discuss our algorithm for three types of PSF. In all the cases, we begin with assuming the energy conservation constraint, which means $H(0, 0,R) = 1$. Thus, s can be solved by noticing the DC components in(5)yields

$$s = \sqrt{Y_0(0,0)/Y_1(0,0)}. \quad (8)$$

A. Gaussian Blur Model

When PSF takes the form of a Gaussian function, we have:

$$H(u,v,R) = exp\{-\frac{1}{2}(u^2 + v^2)\sigma^2\}, \quad (9)$$

Where $\sigma \approx R/\sqrt{2}$. therefore (5) becomes

$$s^2 \frac{|Y_1(u,v)|}{|Y_0(\frac{u}{s}, \frac{v}{s})|} = exp\{-\frac{1}{4}(u^2 + v^2)(R_1^2 - R_0^2/s^2)\}. \quad (10)$$

Above is true for all u, v so an averaged solution is suggested in [1] as well as the following:

$$c \equiv \frac{1}{A_1} \int\int_{I_1} \frac{-4}{u^2 + v^2} ln(s^2 \frac{|Y_1(u,v)|}{|Y_0(\frac{u}{s}, \frac{v}{s})|}) dudv, \quad (11)$$

$$R_1^2 - R_0^2/s^2 = c, \quad (12)$$

where $I1$ is the region within which the integral is well-defined and $A1$ is the area of $I1$. With (7), (8) and (12), we can solve $R0$ and $R1$ uniquely. Here gives a solution with approximation based on the fact that $v \approx f$ and L _ R. We can get the simplified version of (7) as $R1 = sR0$,

so that $R_1^2 - R_0^2/s^2 = R_0^2(s^2 - 1/s^2) = c.$

Hence, $R_0 = \sqrt{\frac{cs^2}{s^4 - 1}}.$

This approximation avoids measuring v,L, f and is found to be accurate enough in experiments.

B. Geometric Blur Model

According to geometric optics, the first order approximation of the PSF takes the form of a cylindrical function in the case of a circular aperture. Therefore we have

$$H(u,v,R) = 2\frac{J_1(R\sqrt{u^2+v^2})}{R\sqrt{u^2+v^2}}. \qquad (13)$$

We adopt the polynomial expansion [6] of a bessel function,

$$J_1(x) = \frac{x}{2} - \frac{x^3}{2^2\cdot 4} + \frac{x^5}{2^2\cdot 4^2\cdot 6} - \frac{x^7}{2^2\cdot 4^2\cdot 6^2\cdot 8} + \ldots, \qquad (14)$$

and (5) becomes

$$s^2\frac{|Y_1(u,v)|}{|Y_0(\frac{u}{s},\frac{v}{s})|} = 1 + a_1(u^2+v^2) + a_2(u^2+v^2)^2 + \ldots,$$

$$a_1 = -\frac{1}{8}(R_1^2 - R_0^2/s^2);\ a_2 = \frac{1}{192}(R_1^4 - R_0^4/s^4) + \frac{R_0^2}{8s^2}a_1; \ldots$$

If we can identify *an*, *n* = 1...*N* , we can solve for *R*0 and *R*1 with (7) and (8). *N* is the number of coefficients we plan to identify. Theoretically, identifying only *a*1 is enough. However more coefficients are desired for a reliable solution. Thus
the identification problem equals solving the matrix equation:

$$\begin{bmatrix} Z(u_0,v_0) \\ Z(u_0,v_1) \\ \ldots \end{bmatrix} = \begin{bmatrix} 1 & u_0^2+v_0^2 & (u_0^2+v_0^2)^2 & \ldots \\ 1 & u_0^2+v_1^2 & (u_0^2+v_1^2)^2 & \ldots \\ 1 & \ldots & \ldots & \ldots \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \ldots \\ a_N \end{bmatrix} \qquad (15)$$

Where $Z(u,v) \equiv s^2\frac{|Y_1(u,v)|}{|Y_0(\frac{u}{s},\frac{v}{s})|}$. The LHS vector is $K \times 1$, K is the number of non-zero frequency components being used.The matrix in RHS is of size $K \times N$, and the vector in RHS is the unknown vector of size $N \times 1$. An least-square solution of [*a*1, ...*aN*] will give us an overdetermined equation array
for solving *R*0 and *R*1.

C. Polynomial Blur Model

When we have no prior knowledge of PSF, we can approximate the OTF by an arbitrary *Mth* order polynomial. Consider:

$$H(u,v,R_0) = 1 + \sum_{n=1}^{M} b_n(u^2+v^2)^n;$$
$$H(u,v,R_1) = 1 + \sum_{n=1}^{M} c_n(u^2+v^2)^n.$$

According to [2], we have a constraint on *b*1:

$$[\frac{\partial^2 H(u,v,R_0)}{\partial u^2} + \frac{\partial^2 H(u,v,R_0)}{\partial v^2}]|_{u=v=0} = 4b_1 = -\frac{R_0^2}{2}.$$

Similar constraint applies to *c*1. One can verify that (13) satisfies this constraint. Therefore, we have

$$\frac{H(u,v,R_1)}{H(u/s,v/s,R_0)} = \frac{1+\sum_{n=1}^{M} c_n(u^2+v^2)^n}{1+\sum_{n=1}^{M} b_n(u^2+v^2)^n/s^n} = 1 + \sum_{n=1}^{N} a_n(u^2+v^2)^n,$$

with

$b_1 = -\frac{1}{8}R_0^2$ and $c_1 = -\frac{1}{8}R_1^2$. $a_n, n = 1, \ldots, N$ can be solved from (15). Here we define

$$\mathbf{b} \equiv [b_1/s, \ldots b_n/s^n, \ldots b_M/s^M]^T, \mathbf{c} \equiv [c_1, \ldots c_n, \ldots c_M]^T;$$
$$\mathbf{a}^{(1)} \equiv [a_1, \ldots, a_n, \ldots, a_M]^T, \mathbf{a}^{(2)} \equiv [a_{M+1}, \ldots, a_N]^T;$$

As long as $N \geq 2M$, a close form solution of **b** and **c** can be given by [7]:

$$\mathbf{c} = -\mathbf{A}^{-1}\mathbf{a}^{(2)}; \qquad \mathbf{b} = \mathbf{c} - \mathbf{K}\mathbf{a}^{(1)}, \qquad (17)$$

Where

$$\mathbf{A} \equiv \begin{bmatrix} a_M & \ldots & a_1 \\ a_{M+1} & \ldots & a_2 \\ \ldots & \ldots & \ldots \\ a_N & \ldots & a_M \end{bmatrix}; \mathbf{K} \equiv \begin{bmatrix} 1 & & & \\ c_1 & 1 & \mathbf{0} & \\ \ldots & \ldots & \ldots & \ldots \\ c_{M-1} & c_{M-2} & \ldots & 1 \end{bmatrix}$$

Once we get the estimation of the transfer function, we can process the degraded image with an inverse filter or a Wiener filter to recover the focused image.

IV. MULTIPLE IMAGE ESTIMATION

All above discussion is based on two frames. We can also use three or more frames for estimation, in order to improve the accuracy and robustness. For instance, in the case of the Gaussian PSF, if we have a third image with spectrum *Y*2(*u*, *v*), we can form another set of equations:

$$s'^2\frac{|Y_2(u,v)|}{|Y_0(\frac{u}{s'},\frac{v}{s'})|} = exp\{-\frac{1}{4}(u^2+v^2)(R_2^2 - R_0^2/s'^2)\}$$
$$s' = \sqrt{Y_0(0,0)/Y_2(0,0)};$$
$$R_2 = s'R_0; \qquad (18)$$

using the simplified version of (7). Along with (10), (8) and *R*1 = *sR*0, we have six equations for five unknowns. It is overdetermined, which enables us to use information from three frames to form one estimation. Define

$$w \equiv \frac{1}{A_2}\int\int_{I_2}\frac{-4}{u^2+v^2}ln(s^2\frac{|Y_1(u,v)|}{|Y_0(\frac{u}{s},\frac{v}{s})|}s'^2\frac{|Y_2(u,v)|}{|Y_0(\frac{u}{s'},\frac{v}{s'})|})dudv; \qquad (19)$$

where the meanings of *A*2 and *I*2 are the same as in (11). The the estimation of *R*0 is then given by

$$R_0 = ss' \sqrt{\frac{w}{s^2(s'^4 - 1) + s'^2(s^4 - 1)}}. \quad (20)$$

As we can see in the simulation results, the estimation based on three images improves the performance of the algorithm.

V. ERROR ANALYSIS

The performance of our estimation algorithm can be evaluated by introducing an additive noise in the model:

$$Y_i(u,v) = H(u,v,R_i)F_i(u,v) + N_i(u,v); \quad i = 0,1. \quad (21)$$

Using the Gaussian blur model and simplified version of (7) as in Section 3.1, we can give the estimation of OTF for first image with presence of noise as:

$$\hat{H}(u,v,R_0) = [s^2 \frac{Y_1(u,v) - N_1(u,v)}{Y_0(u/s, v/s) - N_0(u/s, v/s)}]^{\frac{s^2}{s^4-1}}.$$

The estimation of the focused image is given by ˆ $F0(u,v)$ = $Y0(u, v)$/ˆH (u v,$R0$), while the noise free estimation is $F0(u,v)$ = $Y0(u,v)/H(u,v,R0)$. This makes us arrive at the noisy version of focused image estimate as:

$$\hat{F}_0(u,v) = F_0(u,v)[\frac{Y_1(u,v) - N_1(u,v)}{Y_0(\frac{u}{s},\frac{v}{s}) - N_0(\frac{u}{s},\frac{v}{s})} \cdot \frac{Y_0(\frac{u}{s},\frac{v}{s})}{Y_1(u,v)}]^{\frac{-s^2}{s^4-1}}. \quad (22)$$

Notice that the original additive noise becomesmultiplicative noise in the final estimation. The statistical characteristic of the noise also changes. The random variable inside the square bracket is the ratio of two normal random variables with non-zero mean and its distribution has been studied in [8]. Based on that, we can also give the distribution and expectation of the noise.

VI. CONCLUSIONS

We introduced a novel method for focused image estimation from defocused video sequences. The proposed algorithm relies on the differences in blur characteristic of multiple images resulting from camera motion in video sequences. This notion is exploited to estimate the blur model and obtain a more robust estimation of the focused sequence. The proposed algorithm can be used to correct out-of-focus video sequences as well as replace the expensive auto-focus apparatus in modern cameras.

VII. REFERENCES

[1] M. Subbarao, "Efficient depth recovery through inverse optics", *Machine Vision for Inspection and Measurement (H. Freeman, Ed.), New York: Academic*, 101-126, 1989.

[2] J. Rayala, S. Gupta, and S. Mullick, "Estimation of depth from defocus as polynomial system identification", *IEEE Proceedings*, pp.356-362, 2001.

[3] A. Rajagopalan, S. Chaudhuri, and U. Mudenagudi, "Depth estimation and image restoration using defocused stereo pairs", *IEEE Trans. on PAMI*, vol.26, no.11, pp.1521-1525, Nov. 2004.

[4] J. Yang, D. Schonfeld, C. Chen, and M. Mohamed, "Online video stabilization based on particle filters", *IEEE ICIP*, Atlanta, GA, Nov. 2006.

[5] R. Bracewell, K. Chang, A. Jha, and Y. Wang, "Affine theorem for two dimensional fourier transform", *Electronics Letters*, Feb. 1993.

[6] F. Bowman, "Introduction To Bessel Functions", *Dover publication Inc., New York*, 1958.

[7] D. Graupe, "Time Series Analysis, Identification and Adaptive Filtering", *R.E. Kreiger, Publ. Co.*, 1989.

[8] D. Hinckley, "On the Ratio of Two Correlated Normal Random Variables", *Biometrika*, 1969