Advanced Design and Manufacture of Spiral Bevel, Hypoid, and Worm Gears

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Abstract—A method is presented for the optimization of design and manufacture of spiral bevel, hypoid, and worm gears. The aim of the optimization is to improve the operating characteristics of the gear pair, namely, to increase the service life of gears and to reduce the dynamic effects of gear mesh. Optimal modifications are introduced into the tooth surfaces to simultaneously reduce the maximum tooth contact pressure and transmission errors. These modifications are introduced by applying optimal machine tool settings and geometry of cutting tool in the manufacturing process. The method is demonstrated on spiral bevel, hypoid, and worm gears. The obtained results have shown that significant improvements in the operating characteristics of the gear pairs are achieved.

Keywords—gears, design, manufacture, optimization

I. Introduction

Gear drives are one of the most widely used modes in power transmission system with the advantages of large power, high efficiency, and long service life.

In the gear design process, the traditional method only gives basic parameters of gear structure. The modern power transmission industries need an optimum design of high quality gears as to perform the variation of the operating characteristics of the gears and to reduce the influence of tooth defects and misalignments on tooth mesh, optimal tooth characteristics of gears and to reduce the influence of tooth manufacturing error, and noise.

Nowadays, the optimizing and improving machine tool settings and the geometry of the cutting tool used in the manufacturing procedure. The new CNC machines have made it possible to perform varying correction motions during the manufacturing process.

Spiral bevel, hypoid, and worm gears can be considered as a more general case on gear geometry. That is the reason that these types of gears will be used to demonstrate the optimization procedure of the design and manufacture of gears.

In the last decades many research works have been directed towards the improvements in gear design and manufacture. Because of the limited length of this presentation, only small part of the corresponding publications can be referenced [1-16].

II. Optimal Machine Tool Settings for the Manufacture of Spiral Bevel and Hypoid Gears

In order to improve the operating characteristics of spiral bevel and hypoid gear pairs, modifications are introduced into the teeth surfaces of the pinion. The modifications are introduced by the variation of machine tool settings and the profile of the head-cutter’s cutting edges. The machine tool settings used for pinion tooth finishing are specified in Fig. 1: sliding base setting ($c$), basic radial ($e$), basic offset ($g$), tilt angle ($\beta$), and swivel angle ($\delta$). The other manufacturing parameters are the velocity ratio in the kinematic scheme of the machine tool for the generation of the pinion tooth surface ($i_g$) and the radii of the head-cutter profile ($r_{prof1}$, $r_{prof2}$). The tooth surface of the pinion is defined by the following system of equations:

$$ \bar{r}_1^{(i)} = M_4 (i_g) \cdot M_3 (e, g) \cdot M_2 (\beta, \delta) \cdot \bar{r}_1^{(i)} (r_{prof1}, r_{prof2}) $$

$$ \bar{v}_0^{(r_{prof1})} \cdot \bar{e}_0^{(r_{prof2})} = 0 $$

where $\bar{r}_1^{(i)}$ is the radius vector of head-cutter surface points, matrices $M_1$, $M_2$, $M_3$, and $M_4$ provide the coordinate transformations from system $K_{1T}$ (rigidly connected to the cradle and head-cutter $T_1$) to system $K_T$ (rigidly connected to the being generated pinion). The second equation describes mathematically the generation of pinion tooth surface by the head-cutter; $\bar{v}_0^{(r_{prof1})}$ is the relative velocity vector of the head-cutter to the pinion and $\bar{e}_0^{(r_{prof2})}$ is the unit normal vector of the generator surface of the head-cutter.

The new CNC hypoid generators have made it possible to perform the variation of sliding base setting, basic radial, basic offset, tilt and swivel angles, and the ratio of roll conducted by polynomial functions of fifth-order:

$$ \Delta c = c_{01} + c_{11} \cdot (\psi - \psi_0) + c_{12} \cdot (\psi - \psi_0)^2 \ldots + c_{15} \cdot (\psi - \psi_0)^5 $$

$$ \Delta e = c_{20} + c_{21} \cdot (\psi - \psi_0) + c_{22} \cdot (\psi - \psi_0)^2 \ldots + c_{25} \cdot (\psi - \psi_0)^5 $$

$$ \Delta g = c_{30} + c_{31} \cdot (\psi - \psi_0) + c_{32} \cdot (\psi - \psi_0)^2 \ldots + c_{35} \cdot (\psi - \psi_0)^5 $$

$$ \Delta \beta = c_{40} + c_{41} \cdot (\psi - \psi_0) + c_{42} \cdot (\psi - \psi_0)^2 \ldots + c_{45} \cdot (\psi - \psi_0)^5 $$

(2)

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\[
\Delta \delta = c_{50} + c_{51} \cdot (\psi - \psi_0) + c_{52} \cdot (\psi - \psi_0)^2 + \ldots + c_{55} \cdot (\psi - \psi_0)^5
\]
\[
\Delta \phi_{2,\text{max}} = c_{60} + c_{61} \cdot (\psi - \psi_0) + c_{62} \cdot (\psi - \psi_0)^2 + \ldots + c_{65} \cdot (\psi - \psi_0)^5
\]

where \( \psi \) is the angle of rotation of the cradle in pinion tooth surface generation.

Therefore, the maximum tooth contact pressure and maximum transmission error depend on 32 manufacture parameters:

\[
P_{\text{max}}(mp) = P_{\text{max}} \left( r_{\text{prof}, 1}, r_{\text{prof}, 2}, \sum_{i=1}^{5} \sum_{j=0}^{5} c_{ij} \right)
\]

\[
\Delta \phi_{2,\text{max}}(mp) = \Delta \phi_{2,\text{max}} \left( r_{\text{prof}, 1}, r_{\text{prof}, 2}, \sum_{i=1}^{5} \sum_{j=0}^{5} c_{ij} \right)
\]

An optimization method is applied to systematically define optimal head-cutter geometry and machine tool settings to simultaneously minimize maximum tooth contact pressure and angular displacement error of the driven gear. The goal of the optimization is to minimize tooth contact pressure and transmission errors while keeping the loaded contact pattern inside the physical tooth boundaries of the pinion and the gear. The optimization problem to be solved is as follows:

\[
\min_{mp} f(mp) = \min_{mp} \left[ c_p \cdot \frac{P_{\text{max}}(mp)}{P_{\text{max,0}}} + c_\phi \cdot \frac{\Delta \phi_{2,\text{max}}(mp)}{\Delta \phi_{2,\text{max,0}}} \right]
\]

subject to \( C(mp) = 0 \)

where \( C(mp) \) represents the constraints, \( P_{\text{max,0}} \) and \( \Delta \phi_{2,\text{max,0}} \) are the maximum tooth contact pressure and transmission error obtained for the initial values of manufacture parameters; \( c_p \) and \( c_\phi \) are non-negative weight coefficients, expressing the relative importance of \( P_{\text{max}} \) and \( \Delta \phi_{2,\text{max}} \), respectively.

Functions \( f(mp) \) and \( C(mp) \) are not available analytically, they exist numerically through the load distribution calculation. Therefore, the computer simulation of load distribution must be run, repeatedly, in order to compute the various quantities needed by the optimization algorithm. The load distribution calculation is based on a highly nonlinear system of equations [17]. An iterative technique was used to solve this system of equations. This causes that the calculation of partial derivatives for gradient-based optimization algorithms to be quite impractical. For this reason, a nonderivative method is selected to solve this particular optimization problem. The pattern search method is used.

The CNC machine for generation of spiral bevel and hypoid gears is provided with six degrees-of-freedom for three rotational motions (\( \theta, \zeta, \eta \)), and three translational motions (\( X, Y, Z \), Fig. 2). The six axes of CNC generator are directly driven by the servo motors and able to implement prescribed functions of motions. The face-hobbing method requires simultaneous six-axis control, the face-milling method requires only five-axis control. The following coordinate systems are applied to describe the relations and motions in the CNC generator (Fig. 2): Coordinate systems \( K_i(x, y, z) \) and \( K_i(x, y, z) \) are rigidly connected to the head cutter and the pinion/gear, respectively. The coordinate transformation from system \( K_i \) to system \( K_i \) performs the following equation:
The location and the orientation of the tool with respect to the pinion/gear are given in coordinate systems that are represented for a conventional, cradle-type generator (Fig. 1). An algorithm is developed for the execution of motions on the CNC generator using the relations valid for the cradle-type machine. This algorithm is based on the conditions that the relative position of the axes of the head-cutter and the pinion rotations, \( z_{o0} \) and \( y_{o0} \), and the axial relative position of the head cutter and the pinion/gear should be the same whether the pinion/gear is cut on a cradle-type or a CNC hypoid generator.

To ensure the same relative position of the two axes, \( z_{o0} \) and \( y_{o0} \), on both the cradle-type and CNC hypoid generating machines, the elements of the coordinate transformation matrices should be equal. On the basis of Eqs. (1) and (5) the following condition should be satisfied:

\[
\mathbf{e}_{o0}^{(1o)} = M_{i2} \cdot M_{i1} \cdot M_{c4} \cdot M_{c3} \cdot M_{i2} \cdot \mathbf{e}_{o0}^{(2o)} = M_{i0} (\zeta, X, Y, Z) \cdot \mathbf{e}_{o0}^{(3o)}
\]

The same relative position of the head cutter and the pinion along their axes in the case of both machines, is satisfied by applying the following condition

\[
\mathbf{r}_{o0}^{(1o)} = M_{i2} \cdot M_{i1} \cdot M_{c4} \cdot M_{c3} \cdot M_{i2} \cdot \mathbf{r}_{o0}^{(2o)} = M_{i0} \cdot \mathbf{r}_{o0}^{(3o)}
\]

A computer program was developed to implement the formulation provided above. By applying this program the optimal profile of the head-cutter and the optimal values of machine tool settings were calculated. The main design data of the example hypoid gear pair used in this study are given in Table 1. The transmitted torque was 20 Nm.

<table>
<thead>
<tr>
<th>Design parameters</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>10</td>
<td>41</td>
</tr>
<tr>
<td>Module, mm</td>
<td>3.4</td>
<td></td>
</tr>
<tr>
<td>Pinion offset, mm</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Pressure angle, deg</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Mean spiral angle, deg</td>
<td>52</td>
<td>27.3</td>
</tr>
<tr>
<td>Face width, mm</td>
<td>37.8</td>
<td>31</td>
</tr>
</tbody>
</table>

The load distribution calculation was performed for 21 instantaneous positions of the pinion and the gear rolling through a mesh cycle. The tooth contact pressure distributions along the potential contact lines for 21 instantaneous positions and for all the adjacent tooth pairs engaged for a particular position of the mating members, for the case when no modifications are introduced into the pinion teeth, namely straight-lined head-cutter profile and the basic values of machine tool settings are applied, are shown in Fig. 3. In this case the pinion and gear tooth surfaces are fully conjugate. The obtained maximum tooth contact pressure is 578 MPa and the maximum angular displacement error of the driven gear is 3.06 arcsec. The tooth contact pressure distribution for the case when the pinion teeth are manufactured by the head-cutter of optimized profile and by optimal values of machine tool settings is shown in Fig. 4. It can be observed that the maximum tooth contact pressure is reduced to \( p_{\text{max}} = 300 \text{ MPa} \) and the maximum transmission error to \( \Delta \phi_{2\text{max}} = 2.32 \text{ arcsec} \) .
Improvements in the Precision of Worm Gear Manufacture

The worm gears can be manufactured by a hob or by a flying tool. Because of the finite number of cutting edge of the hob and in the case of the use of a flying tool, geometrical errors will occur on the processed gear tooth surface. The possibilities to reduce these tooth surface errors will be investigated in this section.

The kinematics of worm gear teeth processing by a hob is shown in Fig. 5. The coordinate systems in the case of flying tool are the same as presented in Fig. 5. The gear tooth surface, shaped by the cutting edge of the hob or by the flying tool is defined by the following equation:

\[
\mathbf{r}_g^{(c,e)} = \mathbf{M}_{hg} \cdot \mathbf{r}_n^{(c,e)}
\]  

where matrix \( \mathbf{M}_{hg} \) transforms the coordinates from system \( K_h \) (attached to the hob) to system \( K_g \) (attached to the gear) and \( \mathbf{r}_n^{(c,e)} \) is the position vector of the points of the cutting edge of the hob or of the flying tool.

The gear tooth surface, which will be fully conjugated with the worm surface, is defined by the system of equations:

\[
\mathbf{r}_g^{(s)} = \mathbf{M}_{wg} \cdot \mathbf{r}_n^{(c)}
\]

\[
\mathbf{n}_w^{(c)} \cdot \mathbf{v}_w^{(n,s)} = 0
\]

where matrix \( \mathbf{M}_{wg} \) transforms the coordinates from system \( K_w \) (attached to the worm) into the system \( K_g \) (attached to the gear), \( \mathbf{r}_n^{(s)} \) is the position vector of worm surface points, \( \mathbf{n}_w^{(c)} \) is the normal vector of the worm surface, and \( \mathbf{v}_w^{(n,s)} \) is the relative velocity vector of the worm to the gear.

The calculations were made for the commonly used thread-ground worm gear drive. The investigations have shown that a considerably gear tooth surface error reduction can be achieved by the increase of the number of hob teeth (Fig. 6). But, to ensure the strength of hob teeth and the tool life, the number of hob teeth is usually limited to 6, eventually for larger hob diameters it can be 8, 10, or more.

In the case of worm gear teeth processing by a flying tool, the role of hob tooth number is replaced by the axial shift of the flying tool. It is desirable a relatively small amount of the axial shift of the flying tool (Fig. 7), but by the reduction of this shift the gear tooth processing time and the cost of gear manufacture are increased. A compromise should be found.
iv. Conclusions

Based on the obtained results the following conclusions can be made:

1. In the case of the hypoid gear pair, manufactured by the head-cutter with optimal profile and optimal machine tool settings, considerable reduction in the maximum tooth contact pressure of 48% and in the transmission error of 24% were obtained.

2. In the case of worm gears, the errors of processed gear tooth surface can be considerably reduced by the increase of the number of hob teeth ($N_h$) and by the reduction of the axial shift of the flying tool.

References


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