Non-Asymptotic Real-Time Adaptation to Background Noise in Multichannel C-OTDR Monitoring Systems

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Abstract—A sequential nonparametric method is proposed for adaptation to background noise parameters for real-time. The method is designed to operate as an adaptation-unit, which is included inside a detection subsystem of an integrated multichannel monitoring system. The proposed method guarantees the given size of a nonasymptotic confidence set for noise parameters. Properties of the suggested method are rigorously proved. The proposed algorithm has been successfully tested in real conditions of a functioning C-OTDR monitoring system, which was designed to monitor a railway. (Abstract)

Keywords—Guaranteed estimation, multichannel monitoring systems, non-asymptotic confidence set (key words)

I. Introduction

The detection of targeted noise-like signals observed on the noise-background is topical in multichannel large scale monitoring systems. An example of such a system is brand-new C-OTDR-systems [1]-[4] for monitoring of super-extended objects (oil & gas pipelines, national borders, railways, etc). There is a prior uncertainty about statistical characteristics of noise and signals, and there are tens of thousands of C-OTDR-channels, whose data need to be processed in real-time. An important factor is the dynamics of noise statistical characteristics in different channels. For example, in C-OTDR monitoring systems (CMS) this dynamics depends of the day, the season, and various external factors (technological works on monitoring objects and in its vicinity, vibrations of highways, sounds of underground rivers etc). So at different times and different places the conditions of the observation are dramatically different. These circumstances influence sensors systems very strongly. The influence implies an increase in Type I and Type II errors. While noise level may be dramatically different in various time intervals for one and the same channel, the industrial noise power and its spectral characteristics as a rule are stable for extended periods of time (no less than a few minutes or sometimes even hours). Unlike the industrial noises, targeted signals have high power and a short duration (no more than a few minutes). So, targeted signals have a shorter stability period with respect to the stability period of noises. The important feature of multichannel monitoring systems which are designed for monitoring of super-extended objects is an enormous quantity of channels. The quantity of the channels may be up to 100 000 and more, and channel data have to be processed in real time. In this case, without real-time estimates of the background noise parameters (BN) it is not possible to build a target signal detector which would guarantee prescribed level for Type I and Type II errors. So, there is an objective necessity to design a real-time adaptation procedure (RTAP) which guarantees the quality of the BN-parameter estimation. In this paper it is proposed to find this problem solution from the standpoint of sequential analysis inside of the linear models class. The basic assumption, which has been used in suggested approach, is an existence of a long enough period, which we call the period of the initial system adjustment or ISA-period. During the ISA-period we can observe only noise in each channel of the monitoring system, and we can use this period for an initial adaptation to noise in every channel. During this phase, the parameters of noise are calculated to be used in the subsystem of targeted signals detection. But the channel noise continues to change its characteristics, hence the results of the initial adaptation gradually becomes irrelevant. Because of that, the process of adaptation to noise has to be continued, and we will call this process a “regular adaptation” (RA). During the RA-process, channel observations are used, which do not contain the targeted signals. The time intervals which contain detected signals, are cut out from the channel data stream; and the only remainders are used for adaptation. In CMS, the BN intensity is approximated well by a linear regression with unknown parameters. Those parameters are constant during certain time intervals, which we call as BN-intervals. The BN-intervals durations are enough to build a confidence set with a given size. The set of BN-intervals includes the ISA-period and all time intervals without signals. The main idea is to build the confidence sets for the BN-parameters from the standpoint of sequential analysis. Those sets will be having the given size, and they will be built for finite time. This approach was successfully used for detection of targeted signals in C-OTDR monitoring systems. Systems of that class are new monitoring systems and they are very effective in controlling the
The basis of this method is the use of a vibrosensitive infrared stream injected into a standard monomode fiber by means of a coherent semiconductor laser at the wavelength of 1550 nm. Probing is carried out in the pulsed mode, with the frequency of 8-15 kHz at the pulse length of 20-100 ns. The optical fiber (system sensor) is put into the ground, at the depth of 30-50 cm, at the distance of 5-10 m from the monitoring object. When a pulse is moving along the optical fiber, the Rayleigh elastic backscattering is realized on its natural irregularities, which due to high coherence of the used laser of 3B class leads to formation of the so-called stable interference structures of chaotic type, otherwise called speckles or speckle images. A sequence of speckles is received in the point of emanation using an ordinary welded coupler or a circulator. The central moment of the concept is the phenomenon that any seismic vibration arising on the surface of the optical fiber due to propagation of seismoacoustic waves from the sources of elastic oscillations, changes its local refractive index. Changes of the local refractive index are reflected in the time-and-frequency structure (TFS) of the respective speckle. Knowing the pulse duration and the velocity of wave propagation in the optical fiber, it is easy to determine the section where the TFS speckle deviation took place. Analysis of the sequence of speckle structures using wavelet conversion apparatuses (the phase of singling out of primary signs of target signals) and Lipschitz classifiers (the phase of target signals classification) makes it possible not only to reliably detect the target source of seismoacoustic radiation, but also to determine its type and area of occurrence. In particular, location of the target source of seismoacoustic radiation is determined with the accuracy of up to 5..10 m at the distance of up 40 km from the laser location. Actually, as a result of logical processing, several thousands of the so-called C-OTDR channels are formed on the monitoring distance, each of which transfers information on seismoacoustic activity at the well-defined point of the space. It is obvious that the width of the typical C-OTDR channel is 1..5 m. The following problems are solved in the process of analysis of seismic activity in C-OTDR-systems: a) Target Seismic-Acoustic Events (TSAE’s) detection, [5]; b) TSAE location assessment; and c) TSAE type classification, [6].

Proposed RTAP belongs to the detection subsystem (this subsystem must guarantee the upper bounds for the probability of errors of the first and the second kinds). The RTAP is guaranteed the BS-parameters estimate quality in non-asymptotic sense, hence, BS-parameters guaranteed estimates are a basis for building the TSAE-detection procedure with prescribed characteristics (the detection procedure must guaranteed TSAE detection with given upper bounds for Type I and Type II errors). The proposed algorithm has been successfully tested in real conditions of a functioning of C-OTDR system, which was designed to monitor the ballast of railway tracks. Since the meaning of the RA-process is similar to meaning of the ISA procedure, in this article we will describe the RTAP for the initial phase only.

II. Research Objective

Let us assume that we have a multichannel monitoring system. There are array of channels, which are used for getting targeted signals. Indexes of system channels in conjunction form a set $Z = \{1, 2, ..., z\}$. Observations are made at successive times, which form a set $N = \{1, 2, ...\}$. Thus, the observations are form the following sets $X = \{X(n)\mid n \in N\}$, $X(n) = \{x_1(n), x_2(n), ..., x_z(n)\} \in R^z$, $x_j(n) = \{X(n)\}_j$ is measurement of $j$-th channel at moment $n$.

For each channel $j \in Z$ observations are described by following expressions:

$$\forall n < \tau_j : x_j(n) = a_j^* + \xi_j(n).$$

(1)

Here,

- $\{\xi_j(n)\}$ are random variables with unknown distributions, $E \xi_j(n) = 0$, $E \xi_j^2(n) \leq b_j(n)$,
- $\forall k \neq p \ E \xi_j(k)\xi_j(p) = 0$;
- noise parameters $\theta^* = \{\theta_1^*, \theta_2^*, ..., \theta_z^*\}$ are priori unknown;
- parameters $\{a_j\}, \{b_j\}$ are given.

The research objective is to build the procedure of adaptation to background noise parameters in real time. This procedure has to guarantee the BS-parameters estimate quality in non-asymptotic sense: the prescribed estimation quality has to be achieved while using a finite number of observations only. The solution will be found in form of sequential plan $(r, \delta)$, where $r$ the stopping moment, $\delta = (\delta_1, \delta_2, ..., \delta_z)$ is given z-vector (vector $\delta$ components are prescribed sizes of confidence set $\Xi_j$), and besides

- $P(\tau < \infty) = 1$,
- $\forall \theta_1, \theta_2 \in \Xi_j : \forall j : \left|\theta_1 - \theta_2\right| \leq \delta_j$,
- $P(\theta^* \in \Xi_j) \geq P$;

for prescribed values $\delta_j > 0, P \in (0,1)$.

III. Solution Method

Let us write the equation (1) in following form

$$X(n) = A(n)\theta^* + \xi(n),$$

(1)

where

- $\theta^* = \{\theta_1^*, \theta_2^*, ..., \theta_z^*\} \in R^z$. 


\[ \xi(n) = (\xi_1(n), \xi_2(n), \ldots, \xi_s(n)) \in \mathbb{R}^s, \mathbb{E}\xi(n) = 0, \]
\[ \mathbb{E}\xi(n)\gamma^T(n) \leq \mathbf{B}(n) = \left\| b_\gamma(n) \right\|; \]
\[ \{\mathbf{B}(n)\} \text{ are given sequence of } (z \times z) \text{ matrices}; \]
\[ \{\mathbf{A}(n)\} \text{ are given sequence of } (z \times z) \text{ matrices}; \]
\[ \eta(n) = \sum_{i=1}^{n} \mathbf{A}^T(i)\xi(n). \]
Let us denote:
\[ \Gamma(n) = \left( \sum_{i=1}^{n} \mathbf{A}^T(i)\mathbf{A}(i) \right)^{-1}. \]
\[ c(n) \in \mathbb{R}^s, \forall i : \langle c(n) \rangle > 0. \]
Let the matrices \( \Gamma(n) \) exist for any \( n \). Consider the following expressions:
\[ \theta'(n) = \Gamma(n) \left( \sum_{i=1}^{n} \mathbf{A}^T(i)X(i) + c(n) \right). \]
\[ \theta(n) = \Gamma(n) \left( \sum_{i=1}^{n} \mathbf{A}^T(i)X(i) - c(n) \right). \]
Let us denote:
\[ \theta_i(n) : \langle \forall j : \langle \theta_i(n) \rangle \rangle = \min \left( \langle \theta'(n) \rangle, \langle \theta(n) \rangle \right), \]
\[ \theta_h(n) : \langle \forall j : \langle \theta_h(n) \rangle \rangle = \max \left( \langle \theta'(n) \rangle, \langle \theta(n) \rangle \right). \]
Expressions \( \theta_i(n) \) and \( \theta_h(n) \) are a low bound and an upper bound of a rectangular parallelepiped \( \Xi(n) \in \mathbb{R}^s \). Let us consider matrices:
\[ D(n) = \left[ d_i(n) \right] = \mathbb{E}\eta(n)\eta^T(n) = \sum_{i=1}^{n} \mathbf{A}^T(k)\mathbf{B}(k)\mathbf{A}(k), \]
\[ D^+(n) = \left[ d_i(n) \right] \left( \sum_{i=1}^{n} d_i(n) d_i(n)^T \right)^{-1/2}, n \geq 1. \]
We define the sequence of vectors \{\langle c(n) \rangle\} as follows
\[ c(n) = \left( \mathbf{J}(u^*) / (1 - P) \right)^{0.5} \mathbf{S}(n), \]
where
\[ \mathbf{S}(n) = \left[ d_1(n), d_2(n), \ldots, d_s(n) \right], \]
\[ \mathbf{J}(x) = z^2 \left( x^{0.5} + \left( \frac{z^2 - x}{z^2} \right)^{0.5} \right)^2, \]
\[ u^* = e_z^2 D^+ e_z, e_z = (1, 1, \ldots, 1) \in \mathbb{R}^s. \]
Let us denote
\[ \forall n \geq 1: \pi(n) = 2\Gamma(n)c(n) = \theta_h(n) - \theta_i(n). \]
The sequential plan for estimation of parameter \( \theta^* \) we introduce in following form (\( \pi(n), \tau \)), where \( \tau \) is moment of observations stop:
\[ \tau = \inf \left\{ n \geq 1 | \forall j : \langle \pi(n) \rangle, j \leq \delta_j \right\}. \]

The following theorem describes properties of sequential plan \( (\pi(n), \tau) \).

**Theorem 1.** Let
\[ \lim_{x \to \infty} \Gamma(n)\mathbf{S}^T(n) \to 0. \]

Then
1. \( \mathbf{P}_\theta \left( \tau < \infty \right) = 1. \)
2. \( \mathbf{P}_\theta \left( \delta^* \in \Xi(\tau) \right) = 1, \Xi(\tau) = \{ \theta | \theta \leq \theta^* \}. \)

Proof of Theorem 1 is based on usage Theorem 4.1, p. 510 [5]. So, the confidence rectangular parallelepiped \( \Xi(n) \) with given sizes will be built by the time moment \( \tau \).

The estimation of value \( \mathbb{E} \tau \) (the mean duration of observation in sequential plan \( (\pi(n), \tau) \)) is an important task for practice. We propose a new approach to obtain the upper bound of \( \mathbb{E} \tau \). Let us rewrite (1) in following form:
\[ X(n) = \mathbf{A}(n)\theta^* + \mathbf{G}(n)\epsilon(n). \]
\[ \forall n \geq 1: \xi(n) = \mathbf{G}(n)\epsilon(n), \mathbf{B}(n) = \mathbf{G}^T(n)\mathbf{G}(n), \]
\[ \mathbb{E}\epsilon(n) = 0, \mathbb{E}\epsilon(n)\epsilon^T(n) = \mathbf{E}. \]
\( \mathbf{G}(n) - (m \times m) \) matrix, \( \mathbf{E} \) - the identity \( (m \times m) \) -matrix. Further, we rewrite \( \pi(n) \) in following form:
\[ \forall n \geq 1: \pi(n) = 2\gamma(n) \left( 1 - P \right)^{1/2}, \gamma(n) = \Gamma(n) \left( \mathbf{J}(u^*) \right)^{0.5} \mathbf{S}(n). \]
For some positive constants \( k \) and \( \beta \) we have:
\[ \forall n \geq 1: \mathbf{P}_\theta \left( \left\| \xi(n) \right\| \geq k \left\| \epsilon(n) \right\| \right) = 1, \]
\[ \forall n \geq 1, j : \langle \gamma(n) \rangle_j \leq \beta \left( \sum_{i=1}^{n} \left\| X(i) \right\|^{0.5} \right)^{-1}. \]
For some \( H > 0 \) let us denote
\[ \eta(H) = \inf \left\{ n \geq 1 \left| \sum_{i=1}^{n} \left\| \epsilon(i) \right\| \geq H \right\}, \]
\( \eta(H) \) is the random variable, which have the finite \( \mathbb{E}\eta(H) \).

**Theorem 2.** Let distribution density \( g(z) \) of random variables \( \epsilon(n) \) is a centrally symmetric density. Thus, \( g(z) = g(-z) \), the set \( K(u) = \{ x | g(x) \geq u \} \) is a convex set for any \( u, 0 < u < \infty \). In this case, when (2) and (3) are true for process \( \mathcal{X}(n) \) we have:
\[ \mathbb{E}\tau \leq \mathbb{E}\eta(H) + 1, \]
where
\[ H = \beta \left( \frac{2\gamma \left( \frac{2\gamma \left( 1 - P \right)}{k} \right)^{0.5}}{k} \right) \].
v. Conclusions

Proposed sequential method for adaptation to background noise parameters for real-time is nonparametric (not require to know probabilistic distribution of noises) and this method is non-asymptotic (the required estimation quality is achieved for finite time). The method is designed to operate as an adaptation-unit, which is included inside a detection subsystem of an integrated multichannel monitoring system. Proposed method guarantees the given size of a nonasymptotic confidence set for noises parameters. Properties of the suggested method are rigorously proved. The proposed algorithm has been successfully tested in real conditions of the functioning of C-OTDR monitoring system, which was designed to monitor the railways.

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