MHD natural convection of nanofluid filled trapezoidal enclosure with a stationary adiabatic cylinder

Fatih Selimefendigil, Hakan F. Öztop

Abstract—In this study, MHD free convection of a nanofluid filled trapezoidal cavity with a stationary adiabatic circular cylinder is numerically investigated. The bottom wall of the cavity is heated and the side walls are kept at constant temperature lower than that of the heater. Other walls of the enclosure and cylinder surface are assumed to be adiabatic. The governing equations are solved with a commercial solver using finite element method. The effects of the Grashof number, Hartmann number and solid volume fraction of the nanoparticle are numerically studied for both cylinder and no-cylinder configurations. It is observed that averaged heat transfer increases as the solid volume fraction of the nanoparticle increases. The presence of the cylinder deteriorates the averaged heat transfer and this is more pronounced at low Grashof, high Hartmann numbers and increasing inclination angles of side walls. Averaged heat transfer increases as the solid volume fraction of nanoparticles increases.

Keywords—MHD, nanofluid, obstacle, CFD, finite element

I. Introduction

MHD with nanofluids have received some attention due to higher thermal conductivity of the nanoparticles added to the base fluid [1, 2]. Ghasemi et al. [11] have studied the MHD natural convection in an enclosure filled with water - Al2O3 nanofluid. Their results showed that an enhancement or deterioration of the heat transfer may be obtained with an increase of the nanoparticle volume fraction depending on the value of Hartmann and Rayleigh numbers. Mahmoudi et al. [12] have numerically simulated the MHD natural convection in a triangular enclosure filled with nanofluid. The impact of the Rayleigh number, Hartmann number and nanoparticle volume fraction on the heat transfer and fluid flow are numerically investigated. The studied problem of interest may be encountered in many engineering applications such as solidification, food processing, cooling of electronic devices, coating, glass production, cooling of electronic devices, food processing. The present study aims at investigating the effects of Grashof number, Hartmann number, solid volume fraction of the nanoparticle on the fluid flow and heat transfer for the cases without cylinder and with cylinder located at the center of the trapezoidal cavity.

II. Numerical Model

A schematic description of the physical problem of trapezoidal cavity with an adiabatic cylinder is shown in Fig. 1. A circular cylinder of diameter D = 0.35H is placed at the center of the cavity. No-slip boundary conditions are imposed on the other walls of the cavity and cylinder. The left and right side walls of the cavity are maintained at constant cold temperature of Tc while the bottom wall is at hot temperature of Th. On the other walls of the cavity and on the cylinder surface adiabatic boundary condition is assumed. The cavity is filled with Cu-water nanofluid (different solid volume fractions) under the influence of a vertical magnetic field. The gravity acts in the negative y-direction. The flow inside the cavity is assumed to be laminar, steady and two dimensional. The thermo-physical properties of the fluid are assumed to be constant except for the density variation which is modeled by the Boussinesq approximation in the buoyancy term. The effects of joule heating, displacement currents and induced magnetic field are assumed to be negligible. Thermal equilibrium between the fluid phase and nanoparticles and no slip between them are assumed. The conservation equations of mass, momentum and energy in a 2D Cartesian coordinate is written as:

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\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(1)

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_\text{nf}} \frac{\partial p}{\partial x} + \nu_\text{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  
(2)

\[
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_\text{nf}} \frac{\partial p}{\partial y} + \nu_\text{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \beta_\text{nf} g (T - T_\text{ref}) - \frac{\sigma_\text{nf} \beta_\text{nf} k_\text{nf}}{\rho_\text{nf} c_\text{nf}} B^2 v
\]  
(3)

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_\text{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  
(4)

The dynamic viscosity and thermal conductivity of the nanofluid is given as

\[
\mu_\text{nf} = \mu_\text{f} (1 - \phi)^{n_{25}}, \quad k_\text{nf} = k_\text{f} \left( \frac{k_\text{p} + 2k_\text{f}}{k_\text{p} + 2k_\text{f} + \phi(k_\text{f} - k_\text{p})} \right)
\]  
(5)

Finite element formulation is utilized to solve the Eqs. (1) - (4) using the appropriate boundary conditions described above. The finite element formulation is obtained by establishing the weak form of the governing equations with Galerkin procedure. P2-P1 Lagrange finite elements are used to discretize velocity components and pressure, and Lagrange-quadratic finite elements are chosen for temperature. Segregated parametric solvers are used for fluid flow and heat transfer variables. To obtain an optimal grid distribution, different grid sizes are tested and a grid size with 30108 triangular elements was used. Local and averaged Nusselt numbers are calculated as:

\[
Nu_x = \frac{k_\text{f}}{k_\text{f}} \left( \frac{\partial \theta}{\partial x} \right)_{y=0}, \quad Nu_y = \int_0^y Nu_x \, dx
\]  
(6)

### III. Results and Discussions

Streamlines and isotherms for various Grashof numbers are depicted in Fig. 1 (\(\theta=30^\circ\), Ha=30, \(\phi=0.02\)). At Gr=10^4, two recirculation cells are formed and as the effect of buoyancy is increased with increasing Gr number, the cells are elongated horizontally (Fig. 2 (b-c)) with decreasing values of streamfunction. When the cylinder is placed in the mid of the cavity, flow patterns are disturbed in the vicinity of the cylinder and this effect is more pronounced at low Grashof number. As the Grashof number increases isotherms become parallel to the horizontal walls. Averaged Nusselt numbers versus Grashof numbers for various inclination angles of the side walls are shown in Fig. 3. Averaged heat transfer increases with increasing Gr number and higher values of averaged heat transfer is achieved for decreasing values of inclination angles of the side walls. This is due to the fact that the distance travelled by the fluid particles along the side walls increases with increasing inclination angles. The effect of cylinder on the averaged Nu is shown in Fig. 4. Heat transfer decreases with the presence of the cylinder and this effect is more pronounced at low Gr number and inclination angles of the side walls. Streamlines and isotherms for various Hartmann numbers are shown in Fig. 5 (\(\theta=30^\circ\), Gr=10^5, \(\phi=0.02\)). The strength of the streamfunction decreases as the Hartmann number increases. Isotherms are also affected by the variations in the Ha number. An increase in the Ha number results in isotherms becoming less denser along the bottom wall (decreasing heat transfer) and isotherms become parallel to the side walls with increasing Ha number. Averaged heat transfer decreases with increasing Ha as shown in Fig. 6 since magnetic field retards the velocity and convection. The effect of cylinder on the heat transfer is depicted in Fig. 7. Averaged heat transfer deteriorates with the presence of the cylinder and this effect is more pronounced as the Ha number increases and inclination angle of the side walls decreases. Finally, the effect of nanoparticle volume fraction on the averaged heat transfer is shown in Fig. 8 (Gr=5x10^4, Ha=20). Effective thermal conductivity of nanofluid increases as the volume fraction of the nanoparticles increase which results in better thermal transport of the fluid within the enclosure. Averaged Nusselt number increases as the solid volume fraction of the nanoparticle increases and this trend is the same for all inclination angles of the side walls of the trapezoidal cavity.

### IV. Conclusions

A numerical study of MHD free convection in a nanofluid filled trapezoidal cavity with a stationary adiabatic cylinder was numerically studied. Following conclusions can be drawn from simulation results:

- Heat transfer is enhanced with increasing Grashof number and averaged heat transfer is higher for a square cavity.
- Averaged heat transfer decreases with increasing Hartmann number.
- Adding a cylinder deteriorates averaged heat transfer especially for low Grashof number and high Hartmann number.
- Adding nanoparticles and increasing solid volume fraction of nanofluids enhances averaged heat transfer due to the favorable thermal transport properties of nanofluid.

This study can be extended to include the effects of cylinder rotation angle and transient effects as well.
Fig. 2 Streamlines and isotherms for various Grashof numbers ($\theta=30^\circ$, $Ha=30$, $\phi=0.02$)

Fig. 3 Averaged Nusselt numbers versus Grashof numbers for various angles ($Ha=30$, $\phi=0.02$)

Fig. 4 Difference between averaged heat transfer for cylinder and no-cylinder configurations for various Grashof numbers ($Ha=30$, $\phi=0.02$)

Fig. 5 Streamlines and isotherms for various Hartmann numbers ($\theta=30^\circ$, $Gr=10^5$, $\phi=0.02$)

Fig. 6 Averaged Nusselt number for various Hartmann numbers and inclination angles ($Gr=10^5$, $\phi=0.02$)

Fig. 7 Difference between averaged heat transfer for cylinder and no-cylinder configurations for various Hartmann numbers ($Gr=10^5$, $\phi=0.02$)
Fig. 8 Averaged Nusselt numbers versus nanoparticle volume fraction for various inclination angles (Gr=5x10^4, Ha=20)

References


