A ROM-Less Direct Digital Frequency Synthesizer Based on Bezier Curve Approximation

Ravinder Singh, Kathika Roy and C Bhattacharyya
Defence Institute of Advanced Technology
Pune, India
E-mail: rsd240@yahoo.com, kathikaroy@gmail.com, cbhat0@ieee.org

Abstract—This paper describes the design and implementation of a ROM-Less Direct Digital Frequency Synthesizer (DDS) using Bezier curve approximation. With Bezier curve approximation, phase values between 0 to \(\pi\) are mapped to sine amplitudes. Then, half wave symmetry of sine wave is exploited to construct full sine wave. The proposed approximation introduces maximum error of \(7.9 \times 10^{-4}\), which is equivalent to quantized sinusoid with 16-bit amplitude resolution. This yields very high spectrally pure sine wave output. Based on the approximation, DDS circuit is designed and validated in Matlab-Simulink.

Keywords—Direct Digital Frequency Synthesizer, ROM-Less DDS, Bezier curve approximation.

I. INTRODUCTION

Direct Digital Frequency Synthesizer (DDS) plays a significant role in modern digital communication and electronic instruments. DDS is preferred because of its significant role in modern digital communication and electronic instruments. DDS is preferred because of its significant role in modern digital communication and electronic instruments. DDS is preferred because of its significant role in modern digital communication and electronic instruments. DDS is preferred because of its significant role in modern digital communication and electronic instruments.

II. PRINCIPLE OF DDS

Phase accumulator is \(N\)-bit register. An \(N\)-bit Frequency Control Word (FCW), \(M\) and clock frequency, \(f_{\text{clk}}\), control the phase accumulator output. Phase accumulator output, \(n\) is \(N\)-bit digital sweep whose value ranges from 0 to \(2^N - 1\) and its slope depends upon FCW. These \(2^N\) \((0\text{ to }2^N - 1)\) output values of phase accumulator are mapped to phase values, \(\theta\) as

\[
\theta = 2\pi(n/2^N). \tag{1}
\]

Phase value, \(\theta\) varies from 0 to \(2\pi\). The phase accumulator output is fed to sine generator which gives digital samples of sine wave as

\[
A(n) = \sin(2\pi(n/2^N)). \tag{2}
\]

These digital samples of sine wave are presented to Digital to Analog Converter (DAC). DAC generates an analog waveform in response to the digital samples of sine wave. Generally, a Low Pass Filter (LPF) is placed after DAC to attenuate the higher frequency components. The fundamental frequency, \(f_{\text{out}}\) of output sine wave is [2]

\[
f_{\text{out}} = f_{\text{clk}} \cdot (\text{FCW}/2^N). \tag{3}
\]

Phase to sine amplitude conversion in sine generator is realized by either ROM (look up table) methods or ROM-Less computational methods [3]. Using larger ROM leads to higher power consumption and lower speed, which is undesirable feature for any electronic circuitry. Due to this reason, there is ever increasing demand of keeping memory size to minimum. To achieve it, different ROM compression techniques are employed [4]. The Sunderland technique based upon trigonometric identities, offers a ROM compression ratio of 59:1. Nicholas architecture uses least square approximation to split the ROM into two smaller memories: a coarse one that computes the sine function with high error and a fine one to interpolate the values of coarse ROM to exact value. It achieves a compression ratio of 128:1. Taylor series approximation [5] offers compression ratio of 128:8:1.

An alternative approach, which employs no ROM (look up table) but computes sine samples from phase values is called ROM-Less or computational method. Various techniques fall in this category are CORDIC, Taylor series, parabolic approximation and polynomial approximations. As this approach avoids use of ROM, so guarantees high speed and low power consumption. In this approach, phase values and mapped to amplitudes of sine wave by a sine function. The error in computation of sine amplitude by these different methods is restricted to maximum of \(1.89 \times 10^{-3}\) [6].

In this paper, a new and more accurate, Bezier curve approximation based design for DDS is introduced. The phase values from phase accumulator are mapped to sine amplitudes by Bezier curve approximation. Next section describes the idea of Bezier curve approximation for
generation of sine function. Further on, digital implementation of approximation and its validation is given in detail.

II. BEZIER CURVE APPROXIMATION

A Bezier curve is parametric curve described by polynomial based on control points [7]. A general Bezier equation is written as

\[ B(t) = \sum_{i=0}^{n} b_{i,n}(t)P_i, \quad t \in [0,1] \]

(4)

where the polynomials

\[ b_{i,n}(t) = C_i^n \cdot t^i \cdot (1-t)^{n-i} \]

(5)

\[ C_i^n = \frac{n!}{i!(n-i)!} \]

(6)

and points \( P_i \) are called control points.

A curve is formed by connecting the Bezier points with lines, starting with \( P_0 \), normally not passes through any of other control points and terminating at \( P_n \) is called Bezier curve. In general, every curve can be described by Bezier curves with some minimal error. A Bezier curve with \( n = 6 \) is used to approximate sine function. Equation (7) gives the half cycle of sine wave as

\[ A(t) = (1-t)^6P_0 + 6t(1-t)^5P_1 + 15t^2(1-t)^4P_2 + 20t^3(1-t)^3P_3 + 15t^4(1-t)^2P_4 + 6t^5(1-t)P_5 + t^6P_6 \]

(7)

where \( t \) ranges from 0 to 1. To generate the sine wave having unit amplitude, values of control points \( P_0 \) to \( P_6 \) are calculated and are shown in Table I. \( P_0 \) and \( P_6 \) are kept equal to zero to reduce the terms of Bezier equation. \( P_1 = P_3 \) and \( P_2 = P_4 \) are due to fact that sine wave is symmetrical about \( \pi/2 \).

### Table I. Values of Control Points

<table>
<thead>
<tr>
<th>Control Points</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_0 )</td>
<td>0</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>0.5237</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>1.0464</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>1.3162</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>1.0464</td>
</tr>
<tr>
<td>( P_5 )</td>
<td>0.5237</td>
</tr>
<tr>
<td>( P_6 )</td>
<td>0</td>
</tr>
</tbody>
</table>

As parameter ‘\( t \)’ in (7) varies from 0 to 1 and phase angle ‘\( \theta \)’ varies from 0 to \( \pi \) for first half of sine wave, so replacing ‘\( t \)’ with ‘\( \theta/\pi \)’ and substituting values of control points in (7) leads to

\[
A(t) = \sin(\theta) = 3.14222(\theta/\pi)(1 - (\theta/\pi)) + 15.69544(\theta/\pi)^2
\]

(8)

Further on, a digital sine wave algorithm is used to approximate sine function. Equation (7) gives the curves with some minimal error. A Bezier curve with \( n = 6 \) is used to approximate sine function. Equation (7) gives the half cycle of sine wave as

\[ A(t) = \sin(\theta) = 3.14222(\theta/\pi)(1 - (\theta/\pi)) + 15.69544(\theta/\pi)^2
\]

(8)

Simplification of (8) gives

\[ \sin(\theta) = 3.14222(\theta/\pi) - 0.015669(\theta/\pi)^2 - 5.03539(\theta/\pi)^3 - 0.52663(\theta/\pi)^4 + 3.65319(\theta/\pi)^5 - 1.217743(\theta/\pi)^6. \]

(9)

For phase value, 0 between 0 to \( \pi \), maximum error between real sine wave and proposed approximation is limited to maximum of \( 7.9\times10^{-6} \). Fig. 2 shows the approximation error between real sine wave and sine wave generated by proposed approximation.

![Approximation errors for Bezier curve approximation.](image)

Figure 2. Approximation errors for Bezier curve approximation.

Table II. summarizes the comparison of various sine function approximation techniques along with the proposed approximation. The maximum approximation error in sine amplitude computation by the proposed technique is restricted to maximum of \( 7.9\times10^{-6} \). Employing this approximation in a DDS as sine computation technique, makes it possible to have a ROM-Less DDS architecture with 16-bit amplitude resolution \( (7.9\times10^{-6} < 2^{-16}) \).

### Table II. Summary of Sine Approximation Techniques

<table>
<thead>
<tr>
<th>Approximations Methods</th>
<th>Maximum Error</th>
<th>Equivalent Amplitude Resolution (bits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd-order parabolic approximation [8]</td>
<td>9.0x10^-14</td>
<td>10</td>
</tr>
<tr>
<td>3rd-order parabolic approximation [9]</td>
<td>4.8882x10^-14</td>
<td>11</td>
</tr>
<tr>
<td>5th-order polynomial approximation [10]</td>
<td>2.0x10^-14</td>
<td>12</td>
</tr>
<tr>
<td>8th-order polynomial approximation [11]</td>
<td>1.89x10^-16</td>
<td>15</td>
</tr>
<tr>
<td>Proposed Bezier curve approximation</td>
<td>7.9x10^-16</td>
<td>16</td>
</tr>
</tbody>
</table>
III. DIGITAL IMPLEMENTATION

The phase accumulator output, \( n \) is a digital sweep whose value ranges from 0 to \( 2^N - 1 \). The phase output, \( n \) is related to the phase angle, \( \theta \) as given in (1). Substituting \( \theta \) with \( n \) and multiplying of each term of (9) by \( 1024/2^{10} \) (1024/2^{10} = 1), (9) becomes

\[
\sin(n) = 1/2^{10}(3218(n/2^{N-1}) - 16(n/2^{N-1})^{2} - 5156(n/2^{N-1})^{3} - 539(n/2^{N-1})^{4} + 3741(n/2^{N-1})^{5} - 1248(n/2^{N-1})^{6}).
\]

Considering truncated phase bits \( N=16 \) (as not all bits of phase accumulator are forward to phase to sine conversion stage, but only few MSBs are forwarded and rest are truncated to reduce the ROM requirement. Assuming \( N=16 \) is truncated phase bits to be forwarded for sine function calculation). With \( N=16 \), (10) reduces to

\[
\sin(n) = 1/2^{15}(3218(n/2^{15}) - 16(n/2^{15})^{2} - 5156(n/2^{15})^{3} - 539(n/2^{15})^{4} + 3741(n/2^{15})^{5} - 1248(n/2^{15})^{6}).
\]

Equation (11) can be rewritten as

\[
\sin(n) = n/2^{25}(3218 - n/2^{15}(16 + n/2^{15}(5156 + n/2^{15}(539 - n/2^{15}(3741 - n/2^{15}(1248)))))
\]

The last term in (12), \( n/2^{15}(1248) \) requires a multiplier in its digital implementation, which is not a desirable feature. This term can be replaced with \( n/2^{15}(2^{10} + 2^{3} - 2^{5}) \), which is easily implemented by shift-and-add/subtractor. By this, demand for number of multiplier reduces by one. So (12) can be rewritten as

\[
\sin(n) = n/2^{25}(3218 - n/2^{15}(16 + n/2^{15}(5156 + n/2^{15}(539 - n/2^{15}(3741 - n/2^{15}(1248))))))).
\]

Equation (13) can be realized by digital circuitry. The block diagram based upon (13), for the proposed Bezier curve generator is shown in Fig. 3. It consists of five 15x15-bit multipliers, five shift-and-add scalers, and one adder/subtractor.

Fig. 4 shows the proposed architecture of ROM-Less DDS using Bezier curve approximation. The N-bit phase accumulator generates a digital ramp corresponding to complete cycle of sine wave (0 to \( 2\pi \)). (N−1) LSBS of phase accumulator, corresponding to 0 to \( \pi \) are converted to +ve half cycles of sine wave by Bezier curve approximation generator. The frequency of these half cycles is twice of output frequency. The MSB is used to determine whether the sine amplitude output has to be inverted. Every alternative half cycle is inverted to make complete sine wave.

![Figure 4. Architecture of Bezier curve approximation based DDS.](image)

Table III. Specification of the simulated DDS

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCW</td>
<td>32-bit</td>
</tr>
<tr>
<td>Phase accumulator output</td>
<td>16-bit</td>
</tr>
<tr>
<td>Clock frequency</td>
<td>1GHz</td>
</tr>
<tr>
<td>Output frequency</td>
<td>10MHz</td>
</tr>
<tr>
<td>Spurious Free Dynamic Range (SFDR)</td>
<td>-99.76dBc</td>
</tr>
</tbody>
</table>

The Fig. 5 shows the output of phase accumulator and sine function generator. The output waveform is spectrally very pure. Fig. 6 shows the power spectrum of waveform synthesized by DDS based on Bezier curve approximation. The SFDR evaluated for the proposed method is -99.76dBc.

![Figure 3. Block diagram of Bezier curve approximation](image)
V. CONCLUSION

The DDS using Bezier curve approximation has been proposed and simulated. It avoids use of ROM (look up table) and computes the sine function. The proposed approximation introduces maximum error of $7.9 \times 10^{-6}$, which is equivalent to quantized sinusoid with 16-bit amplitude resolution. The SFDR of proposed ROM-Less DDS is evaluated using Matlab-Simulink, which is equal to $-99.76$dBc. Important advantage lies with this idea of approximation is that it uses full digital circuitry (no memory), gives 16-bit resolution and spectrally very pure sine waveform.

REFERENCES