Large Scale Model Order Reduction of Linear Time Invariant Dynamic System by Improvement of Generalised Least Squares Method

G.Vasu¹
Assistant professor, Dept. of Electrical Engineering
S.V.P Engineering College
Visakhapatnam, A.P., India
E-mail: vasuganjiki@yahoo.com

P.Murari²
Assistant professor, Dept. of Electrical Engineering
S.V.P Engineering College
Visakhapatnam, A.P., India
E-mail: murarinitt@gmail.com

D.S.R.S.Chandra Mouli³
B.E. student, Dept of Electrical
S.V.P Engineering College
Visakhapatnam, A.P., India
E-mail: moulidsrs@gmail.com

Abstract: This paper presents an improvement to generalised least squares method of model order reduction. The improvement enhances the flexibility of the method with very little computational requirement. The reduction procedure is simple, efficient and always generates stable reduced models for the stable high order systems. The proposed method is illustrated with typical numerical examples taken from the literature and the results are compared with the other existing methods to show its superiority.

Keywords: Improved least squares method, Integral Square Error, Order reduction, Moment Matching.

I. INTRODUCTION

The mathematical description of most physical systems leads to higher order differential equations which are difficult to use either for analysis or controller synthesis. It is hence useful and sometimes necessary to find the possibility of finding some equation of the same type but of lower order that may be considered to adequately reflect the dominant characteristics of the system under consideration. Numerous methods are available in the literature for order reduction of linear continuous systems in time domain as well as in frequency domain [1]-[3]. Basing on the simplicity and amicability the frequency domain dependent methods have become more prominent. Transfer function reduction methods are one of the important groups in the frequency domain category.

A popular approach, known as Pade approximation method[4]-[5] for deriving reduced order models has been based on matching of the time moments of original and reduced order Systems. A serious drawback of this method is generating unstable reduced model for a stable higher order model. To overcome this problem Shoji et all[6] suggests using a least squares time moment to obtain a reduced transfer function denominator, and numerator by exact time moment matching. This method has been refined by Lucas and Beat[7], in which the linear shift point was about general point ‘a’, where a=(1-α) and a is the real part of the smallest magnitude pole. Further the method of model order reduction by least squares moment matching was generalised [8] by including the markov parameters in the process to cope with a wider class of transfer function. On the other hand, Aguirre [9] has argued that one of the chief advantages of the least squares Pade (LS-Pade) method is that additional information concerning the original system over the mid-frequency range is included in the simplified model, and consequently better approximations are often obtained. Recently Parmar and Prasad et all[10] entendedthe concept of order reduction by least squares moment matching and generalised least squares methodsabout general point ‘a’ in order to have better approximations. Some heuristic criteria was employed for selecting the linear shift point ‘a’, based upon the means (arithmetic, harmonic and geometric) of real parts of the poles of higher order system. In spite of the significant number of methods available, no approach always gives the best results for all systems. Almost all methods, however aim at accurate reduced models for a low computation cost. In addition it is desired to preserve the stability of the original model; i.e., given a stable high order model, the reduced order model should also be stable.

To overcome these difficulties in this paper, the concept of order reduction by generalised least squares method has been improved in order to have better approximations of high order linear time-invariant dynamic systems for a low computational cost. The relative mapping errors between the original and reduced models are determined, time and frequency responses are plotted to show the effectiveness of the method.
II. DESCRIPTION OF PROPOSED METHOD

Let the transfer function of the original high order linear dynamic SISO system of order ‘n’ be:

\[ G_n(s) = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s + \cdots + b_{n-1} s^{n-1}}{a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + a_n s^n} \quad (1) \]

And let the corresponding \( r \)-order reduced model is synthesized as:

\[ G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{d_0 + d_1 s + \cdots + d_{r-1} s^{r-1}}{e_0 + e_1 s + \cdots + e_{r-1} s^{r-1} + s^r} \quad (2) \]

Further, the method consists of the following steps.

Step 1: Determination of the denominator coefficients of reduced order model : Expand \( G_n(s) \) about \( s=0 \), to obtain the time moment proportional \( (c_i) \) are given by

\[ G_n(s) = c_0 + c_1 s + c_2 s^2 + \cdots \quad \ldots (3) \]

\[ = \sum_{i=0}^{\infty} c_i s^i \]

Similarly Expand \( G_n(s) \) about \( s=\infty \), to obtain the markov parameters \( (M_j) \) are given by:

\[ G_n(s) = M_1 s^{-1} + M_2 s^{-2} + M_3 s^{-3} + \cdots \quad \ldots (4) \]

\[ = \sum_{j=1}^{\infty} M_j s^{-j} \]

Evaluating Eq.(2) and(3) to retain ‘t’ Time moments of the original model in the reduced model gives the following set of equations:

\[
\begin{align*}
    d_0 &= e_0 c_0 \\
    d_1 &= e_1 c_0 + e_0 c_1 \\
    &\vdots \\
    d_{r-1} &= e_{r-1} c_0 + \cdots + e_0 c_{r-1} \\
    \end{align*} \quad (5)
\]

\[
\begin{align*}
    -c_0 &= e_{r-1} c_1 + \cdots + e_1 c_{r-1} + e_0 c_r \\
    -c_1 &= e_{r-1} c_2 + \cdots + e_1 c_{r-1} + e_0 c_{r+1} \\
    \vdots \quad \cdots (6)
    \\
    -c_{r-1} &= e_{r-1} c_{r-1} + \cdots + e_0 c_{r-2} \\
    \end{align*}
\]

Evaluating Eq.(2) and(4) to retain ‘m’ Markov parameters of the original model in the reduced model gives the following set of equations:

\[
\begin{align*}
    d_{r-1} &= M_1 \\
    d_{r-2} &= M_1 e_{r-1} + M_2 \\
    &\vdots \\
    d_0 &= M_1 e_1 + M_2 e_2 + \cdots + M_r e_r \\
    \end{align*} \quad (7)
\]

\[
\begin{align*}
    M_1 e_0 + M_2 e_1 + \cdots + M_r e_{r-1} &= -M_{r+1} \\
    \vdots \\
    M_m e_0 + \cdots + M_{m-1} e_{r-1} &= -M_m \\
    \end{align*}
\]

Step 2: Elimination of \( d_j \) \((j=0,1,\ldots ,r-1)\) in Eq.(7) by substituting Eq.(5) gives the reduced denominator coefficients as the solution of:

\[ e = [P^T P]^{-1} P^T q \quad \ldots (9) \]

Step 4: Finally the reduced denominator is obtained as:

\[ D_r(s) = e_0 + e_1 s + \cdots + e_{r-1} s^{r-1} + s^r \quad \ldots (10) \]

Step 5: Hence from Eq(5) the reduced numerator polynomial is obtained as:

\[ N_r(s) = d_0 + d_1 s + \cdots + d_{r-1} s^{r-1} \quad \ldots (11) \]

III. RELATIVE MAPPING ERRORS

The relative mapping errors of the original model relativeto its Reduced model are expressed by means of the relative integral square error criterion, which are given by [11] :

\[ I = \int_0^{\infty} [H(t) - H_r(t)]^2 . dt / \int_0^{\infty} H^2(t) . dt \ldots (12) \]

\[ J = \int_0^{\infty} [G(t) - G_r(t)]^2 . dt / \int_0^{\infty} [G(t) - G(z)]^2 . dt \ldots (13) \]

Where, \( H(t) \) and \( G(t) \) are the impulse and step responses of original system, respectively, and \( H_r(t) \) and \( G_r(t) \) are that of their approximants.

IV. NUMERICAL EXAMPLES

Two numerical examples are chosen from the literature to show the flexibility and effectiveness of the proposed reduction algorithm than other existing methods, and the response of the original and reduced models are compared.

Example-1: Let us consider the system described by the transfer function [12]:

\[ G_4(s) = \frac{14s^3 + 248s^2 + 900s + 1200}{s^4 + 18s^3 + 102s^2 + 180s + 120} \quad \ldots (14) \]
For which a second order reduced model \( R_2(s) \) is desired.

**Step-1:** Expand \( G_s(s) \) about \( s=0 \), gives the time moment proportional’s \( C_i \) where \( i=0,1,2,\ldots \) which are shown below.

\[
G_s(s) = 10 - 7.5s + 4.81667s^2 - 2.2333s^3 + \ldots \quad \text{...(15)}
\]

Similarly, Expand \( G_s(s) \) about \( s=\infty \) gives the markov parameters \( M_j \) where \( j=1,2,3,\ldots \) which are shown below

\[
G_s(s) = 14s^{-1} - 4s^{-2} - 456s^{-3} - 7296s^{-4} + \ldots \quad \text{...(16)}
\]

**Step-2:** Taking \( t=4 \) time moment proportional's and \( m=0 \) markov parameters of \( G_s(s) \) in Eq.(8) gives the reduced denominator coefficients as the solution of:

\[
\begin{bmatrix}
C_3 \\
C_2 \\
C_1
\end{bmatrix}
\begin{bmatrix}
\delta_0 \\
\delta_1
\end{bmatrix} =
\begin{bmatrix}
-\delta_0 \\
-\delta_0
\end{bmatrix} \quad \text{...(17)}
\]

i.e.
\[
\begin{bmatrix}
-2.2333 \\
4.81667 \\
-7.5
\end{bmatrix}
\begin{bmatrix}
\delta_0 \\
\delta_1
\end{bmatrix} =
\begin{bmatrix}
7.5 \\
10
\end{bmatrix} \quad \text{...(18)}
\]

**Step-3:** The reduced denominator coefficients from Eq.(18) are obtained using Eq.(9): Therefore,

\[
D_r(s) = s^2 + 2.138151s + 1.253176 \quad \text{...(19)}
\]

**Step-4:** After obtaining the reduced denominator the numerator coefficients are determined by matching the first \( (r-1) \) time moments of the system to the reduced model via the first \( (r-1) \) of Eq.(5).

Therefore finally 2\textsuperscript{nd} order reduced model is obtained as:

\[
R_2(s) = \frac{11.982691s + 12.531758}{s^2 + 2.138151s + 1.253176} \quad \text{...(20)}
\]

The proposed method produces quite different reduced models gives the results as shown in Table I, where ‘t’ time moments and ‘m’ markov parameters are used to calculate the denominator and the numerator is determined by matching the first \( (r-1) \) time moments of the original system.

**TABLE I**
COMPARISON OF SECOND ORDER MODELS BY PROPOSED

<table>
<thead>
<tr>
<th>t</th>
<th>m</th>
<th>( d_0 )</th>
<th>( d_1 )</th>
<th>( e_0 )</th>
<th>( e_1 )</th>
<th>I(%)</th>
<th>J(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>12.5317</td>
<td>11.9826</td>
<td>1.2531</td>
<td>2.1381</td>
<td>1.338</td>
<td>0.12</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>3.9063</td>
<td>3.1775</td>
<td>0.3906</td>
<td>1.6647</td>
<td>1.148</td>
<td>0.52</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>8.5523</td>
<td>13.4671</td>
<td>0.8552</td>
<td>1.9881</td>
<td>1.046</td>
<td>0.39</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>10.8898</td>
<td>13.2789</td>
<td>1.0889</td>
<td>2.1446</td>
<td>1.016</td>
<td>0.33</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>11.9266</td>
<td>13.1593</td>
<td>1.1926</td>
<td>2.2104</td>
<td>1.012</td>
<td>0.29</td>
</tr>
</tbody>
</table>

A comparison of the proposed algorithm with the other well known existing order reduction techniques for a second-order reduced model, is given in Table II and the values of I and J are comparable for the proposed and the other existing.
Example-2: Consider a fifth order system described by the transfer function [17]:

\[ G_5(s) = \frac{10s^4 + 82s^3 + 264s^2 + 369s + 156}{2s^5 + 21s^4 + 84s^3 + 173s^2 + 148s + 40} \]  

By using proposed method gives the result as shown below, where \( t=3 \) time moment proportional’s and \( m=1 \) markov parameters of \( G_5(s) \) are used in Eq.(8) and Reduced denominator coefficients from Eq.(9) are obtained as:

\[ D_r(s) = s^2 + 3.092422s + 1.356474 \]  

By matching \((r-1)\) time moments of the original system to the reduced model via the first \((r-1)\) of Eq.(5) the denominator coefficients are obtained as:

\[ N_r(s) = 5s + 5.290247 \]  

Therefore, finally \( R_2(s) \) is given as:

\[ R_2(s) = \frac{5s + 5.290247}{s^2 + 3.092422s + 1.356474} \]  

The second order models generated to Eq. (21) gives the results as shown in Table III, for different ‘\( t \)’ time moments and ‘\( m \)’ markov parameters are used to calculate the denominator and the numerator is determined by matching the first \((r-1)\) time moments of the original system to the reduced model via the first \((r-1)\) of Eq.(5).

**TABLE III**

<table>
<thead>
<tr>
<th>( t )</th>
<th>( m )</th>
<th>( d_0 )</th>
<th>( d_1 )</th>
<th>( e_0 )</th>
<th>( e_1 )</th>
<th>( I (%) )</th>
<th>( J (%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>5.2902</td>
<td>5</td>
<td>1.3564</td>
<td>3.0924</td>
<td>0.0171</td>
<td>0.00311</td>
</tr>
</tbody>
</table>
A comparison of the proposed algorithm with the other well known existing order reduction techniques for a second-order reduced model is given in Table IV. Figure 4(d)-(f) Presents diagrams of step, impulse and frequency responses of $G_{5}(s)$ and $R_{2}(s)$ respectively.

### Table IV

<table>
<thead>
<tr>
<th>Method of Reduction</th>
<th>Reduced Models</th>
<th>I (%)</th>
<th>J (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method</td>
<td>$\frac{5s + 5.290247}{s^2 + 3.092422s + 1.35647}$</td>
<td>0.0171</td>
<td>0.0031</td>
</tr>
<tr>
<td>GLSM[8] (t=3; m=1)</td>
<td>$\frac{5s - 12.730345}{s^2 + 5.638491s + 3.264191}$</td>
<td>1.1121</td>
<td>0.4671</td>
</tr>
<tr>
<td>GLSM[8] (t=4; m=1)</td>
<td>$\frac{4.47315s + 8.686491}{s^2 + 4.119558s + 2.227305}$</td>
<td>0.9799</td>
<td>3.238</td>
</tr>
<tr>
<td>GLSM[8] (t=3; m=2)</td>
<td>$\frac{4.012139s + 9.388536}{s^2 + 4.241596s + 2.407317}$</td>
<td>2.324</td>
<td>0.7318</td>
</tr>
<tr>
<td>PSO[13]</td>
<td>$\frac{347.0245s + 225.6039}{135.68s^2 + 166.381s + 57.84}$</td>
<td>11.766</td>
<td>3.5178</td>
</tr>
<tr>
<td>Mixed DE &amp; CFE [18]</td>
<td>$\frac{369s + 156}{93.5606s^2 + 151.1163s + 39}$</td>
<td>1.6205</td>
<td>0.3388</td>
</tr>
</tbody>
</table>

### V. CONCLUSION

The concept of order reduction by generalised least squares method has been improved and employed to determine the reduced denominator polynomials and numerator polynomials are obtained by matching initial time moments of the original model. The proposed method generates better approximations of a higher order linear, time invariant dynamic systems. The relative step and impulse mapping errors between the original and reduced order models are also determined and plotted with respect to time. A comparison of these mapping errors for the proposed reduction method and the other well known existing order reduction techniques is also given, from which it is clear that the proposed method compares well with the other existing techniques. The results show that the proposed method leads to good and stable reduced models for linear time invariant systems.
REFERENCES


